

AN
ILLUSTRATION AND MENSURATION
OF
SOLID GEOMETRY;

IN SEVEN BOOKS:

CONTAINING

FORTY-TWO moveable Copper-Plate Schemes for forming the various Kinds of Solids, and their Sections; by which the Doctrine of Solids in general, and those in the Eleventh, Twelfth, and Fifteenth Books of EUCLID are elucidated, and rendered more easy to Learners than by any Work hitherto published.

B O O K

- | | |
|--|--|
| I. Contains the five regular Solids. | IV. Contains fundry Sorts of Prisms. |
| II. Shews the Inscription and Circumscription thereof, as set forth in the Fifteenth Book of the Elements. | V. Various Kinds of Pyramids, and Frustums thereof. |
| III. Exhibits a great Variety of irregular Solids. | VI. Some difficult Propositions in the Eleventh and Twelfth Books. |
| | VII. The Cone and its several Sections. |

BY THE LATE

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THE THIRD EDITION.

REVISED, CORRECTED, AND AUGMENTED, BY

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L O N D O N :

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P R E F A C E.

HAVING accidentally purchased the plates of the following work, its being out of print, and very scarce, and the many importunities I have had for it from several friends and customers, are the principal reasons that have induced me to prepare and send it forth again to the public.

Probably in no branch of the mathematics does the student meet with more embarrassment than in the study of the Geometry of Solids; for most of the authors who have treated thereon have adopted the usual method of copying the abstruse and perplexed linear schemes of the ancients, which for the greater part are so inconsistently delineated in perspective, as to render it impossible, even to Euclid himself, to divine the bodies intended to be represented by them. It was, undoubtedly, from a consideration of this kind, that Mr. Cowley was excited to an attempt of exhibiting the various kinds of Solids treated of by Euclid, and other Geometricians, by moveable and folding paper schemes or figures, by which a person of the most slender capacity should at a small, or no expence of time, have a clear and rational idea of the several Solids, and

thereby be able to investigate their properties with more perspicuity and precision.

To Teachers of the Mathematics, Artificers, and other mechanics, this work may be considered of very great utility; as in respect to the first, it may be said to do more service to their young pupils than all the tedious directions and remarks that could possibly be given them: and to the advantage of the latter; the measurement of the several kinds of bodies, which, from a perfect comprehension of their nature and properties, must be attended with more certainty and satisfaction to the mind, than by that dark and groveling method, the rule only.

Mr. Cowley, in the former edition, gave this work the title of "An Appendix to Euclid's Elements;" consequently the student of that great Geometrician, will, by the occasional references met with in this work, clearly understand what part of the above valuable author is intended to be illustrated.

I shall conclude with Mr. Cowley's account of the order and contents of the following pages.

" As the title page informs, I have divided this appendix
 " into seven books, each of which is complete on the sub-
 " ject whereof it treats. In the first is contained the five
 " regular bodies, with the rules for finding either their su-
 " perficial or solid content.

" The second book contains an entire new and exact
 " method of exhibiting to view the actual inscription of one
 " regular

“ regular solid in another, which explains most intelligibly
“ all the propositions of Euclid’s fifteenth book.

“ In the third is exhibited a variety of irregular bodies,
“ such as the octoedron, or, as workmen commonly call it,
“ the canted cube, the octoexaedron, the tetraexaedron, the
“ triexoctaedron, the hexaoctaedron, solid rhombus, solid
“ rhomboides, dodecarhombus, &c.

“ The fourth contains the various sorts of prisms, as the
“ triangular, quadrangular, pentangular, hexangular, octan-
“ gular, &c.

“ The fifth contains the various kinds of pyramids, and
“ their frustrums.

“ In the sixth, the propositions of Euclid’s eleventh and
“ twelfth books are clearly explained, and so illustrated, as
“ renders them perfectly easy to be comprehended, without
“ perplexing the mind to conceive lines as drawn through
“ such and such points, as raised up and meeting together
“ at such and such points above the plane, having only an
“ imaginary existence in the mind that conceives them; for
“ the reader has here each solid really formed, and is at the
“ same time furnished with an ocular view of its sections,
“ justly made, and laid open to his sight: wherefore, by
“ the peculiar contrivance here made use of, there is no
“ longer any difficulty in perceiving, in the most clear and
“ convincing manner, that a triangular pyramid is the third
“ part of a triangular prism, having each the same base and
“ altitude, as set forth in Euclid’s twelfth book, prop. 7;
“ for

“ for the prism is here absolutely divided into three such
“ equal pyramids, each of which appears distinctly, as also
“ the whole prism itself, and, in general, that every pyra-
“ mid is the third part of a prism, having the same base
“ and altitude ; and the like evidence is also produced of
“ the other propositions, &c.

“ Lastly ; the seventh contains the cone and its sections,
“ each being really formed : whereby it evidently appears,
“ that in cutting the cone perpendicularly through its axis,
“ the section is an isosceles triangle ; when cut parallel to
“ its base, the section is a circle ; when cut obliquely, the
“ section is an ellipsis ; when cut parallel to one of the
“ sides of the cone, the section is a parabola ; and when
“ cut parallel to its axis, that the section is an hyperbola.”

WILLIAM JONES.

Holborn,

June 20th, 1787.

A N.

A N
ILLUSTRATION AND MENSURATION
O F
SOLID GEOMETRY.

B O O K I.

*Of the five regular Solids, commonly called
the Platonic Bodies.*

SOLIDS, or bodies, are figures having length, breadth, and thickness.

A *regular body* is a solid contained under a certain number of similar and equal plane figures.

The whole number of regular bodies which can possibly be formed are five; to form which, as they are described in this book,

Raise up the moveable parts of the scheme, and fold them around the part that is shaded.

I. *The Tetraedron, or regular Pyramid, Plate 1,*

Is a solid; bounded by four equilateral triangular faces.

II. *The Hexaedron, or Cube, Plate 2,*

Which is bounded by six equal square faces.

III. *The*

Of the five regular Solids.

III. *The Octaedron, Plate 3,*

Which is bounded by eight equal triangular faces, and is a double tetraedron.

IV. *The Dodecaedron, Plate 4,*

Which is bounded by twelve equal pentagonal faces.

V. *The Icofaedron, Plate 5,*

Which is bounded by twenty equal triangular faces.

There are particular rules for finding either the superficial or solid content of each of those bodies to be met with in the common books on mensuration; but the same may be more readily obtained by the following table:

Surfaces and Solidities of the regular Bodies.			
No. of Sides.	Names.	Surfaces.	Solidities.
4	Tetraedron.	1,732051	0,1178511
6	Hexaedron.	6,000000	1,0000000
8	Octaedron.	3,464102	0,4714045
12	Dodecaedron.	20,645729	7,663119
20	Icofaedron.	8,660254	2,181695

Use of the above Table.

To find the superficial content, multiply the tabular number standing under surfaces, right against the respective body, by the square of the side given: and for the solid content, multiply the tabular number, which stands accordingly, by the cube of the given side.

EXAMPLE

Of the five regular Solids.

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EXAMPLE I. Plate 1.

What is the content superficial and solid of a tetraedron, each linear edge, or side, being 9 inches?

<i>Superficies.</i>	<i>Solidity.</i>
1,732051	0,1178511
81	729
1732051	10606599
13856408	2357022
Area 140,296131 Sup.	8249577
	Area 85,9134519 Solid.

EXAMPLE II. Plate 2.

What is the content, superficial and solid, of a hexaedron, or cube, each side being 9 inches?

<i>Superficies.</i>	<i>Solidity.</i>
81	729
6	1
Area 486 Sup.	Area 729 Solid.

EXAMPLE III. Plate 3.

What is the content, superficial and solid, of an octaedron, each side being 9 inches?

<i>Superficies.</i>	<i>Solidity.</i>
3,464102	,4714
81	729
3464102	42426
27712816	9428
Area 280,592262 Sup.	32998
	Area 343,6506 Solid.

B

EXAMPLE

*Of the five regular Solids.*EXAMPLE IV. *Plate 4.*

What is the content, superficial and solid, of a dodecaedron, each side being 9 inches?

<i>Superficies.</i>	<i>Solidity.</i>
20,645729 81	7,663119 729
20645729 165165832	68968071 15326238 53641833
Area 1672,304049 Sup.	Area 5586413751 Solid.

EXAMPLE V. *Plate 5.*

What is the content, superficial and solid, of an icosaedron, each side being 9 inches?

<i>Superficies.</i>	<i>Solidity.</i>
8,660254 81	2,181695 729
8660254 69282032	19635255 4363390 15271865
Area 701,480574 Sup.	Area 1590,455655 Solid.

End of the first Book.

BOOK

B O O K II.

*Containing an Illustration of the fifteenth Book of
EUCLID's Elements.*

Directions for folding the schemes together.

P L A T E VI.

FIRST form the tetraedron, by folding together its several triangles around that particular one which is shaded.

Then move its vertex till the other angular point, which is moveable, is so much elevated above the plane, that the side which connects those two points of the tetraedron becomes parallel to the plane from which the whole was raised.

The tetraedron being in that position, bring the several parts which constitute the hexaedron around it; thus will the four angular points of the tetraedron fall exactly in the four angles of the cube.

P L A T E VII.

The octaedron being first formed, fold the tetraedron over it.

P L A T E VIII.

First form the octaedron, and raise it perpendicularly upon one of its angular points, then fold the cube around it, and the angular points of the octaedron will then touch the center of each side of the cube.

P L A T E IX.

Form the cube, and incline it a little forward till its sides bisect those of the octaedron, when folded around it.

P L A T E X.

Form the dodecaedron, and incline it so that its sides may bisect those of the icosaedron, when folded around it.

End of the second Book.

B O O K III.

Containing several irregular Solids.

To fold any of the figures contained in this book,

BRING the contiguous parts of each figure around that which is shaded; the rest will then join together, and form the solid required.

P R O B L E M.

To measure an irregular solid of any form whatsoever.

G E N E R A L M E T H O D.

Procure a suitable vessel, as a tub, cistern, or any such as can most easily be measured; then put the body whose content is required into it, and pour into the vessel so much water as will just cover the solid: mark the side of the vessel where it is even with the surface of the water, then take out the solid, and particularly observe how much the water descends; for that cubical part of the vessel contained between the highest ascent and lowest descent of the water being measured, gives the solidity of the body required.

E X A M P L E.

Suppose the solidity of a very irregular piece of stone was required, and that having put it into a *square* cistern, whose dimensions are 50 inches, after having just covered it with water, on taking it out I find the water to descend 8 inches, from thence to find its content.

50 inches,
50

Superficial area, 2500 of the base of the cistern,
8 its depth, or descent of the water.

Solidity, - 20,000 of that part of the cistern possessed by the body, and therefore the content of the body required in inches, which divided by 1728, gives 11,516 feet, &c. the content in solid feet.

N. B. To put the water into the vessel first, and then immerse the solid therein, may, in some cases, be a more preferable way of proceeding.
The

Of irregular Solids.

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The more general method of finding the magnitude and weight of bodies is by means of what is called their *Specific Gravities*.

The specific gravities of bodies are their relative weights contained under the same given magnitude, as a cubic foot, or a cubic inch, &c.

The specific gravities of several sorts of bodies are expressed by the numbers annexed to their names in the following table.

A table of the specific gravities of bodies.

Fine gold - - - - -	19640	Brick - - - - -	2000
Standard gold - - - - -	18888	Light earth - - - - -	1984
Quicksilver - - - - -	14000	Solid gunpowder - - - - -	1745
Lead - - - - -	11325	Sand - - - - -	1520
Fine silver - - - - -	11091	Pitch - - - - -	1150
Standard silver - - - - -	10535	Dry box wood - - - - -	1030
Copper - - - - -	9000	Sea water - - - - -	1030
Gun metal - - - - -	8784	Common water - - - - -	1000
Cast brass - - - - -	8000	Dry oak - - - - -	925
Steel - - - - -	7850	Gunpowder, shaken - - - - -	922
Iron - - - - -	7645	Dry ash - - - - -	800
Cast iron - - - - -	7425	Dry maple - - - - -	755
Tin - - - - -	7320	Dry elm - - - - -	600
Marble - - - - -	2700	Dry fir - - - - -	550
Common stone - - - - -	2520	Cork - - - - -	240
Loom - - - - -	2160	Air - - - - -	1,25

Note.—The several sorts of wood are supposed to be dry; also a cubic foot of water weighs just 1000 ounces, or $62\frac{1}{2}$ pounds avoirdupois. The numbers in this table express not only the specific gravities of the several bodies, but also the weight of a cubic foot of each, in avoirdupois ounces; and thence, *by proportion*, the weight of any other quantity, or the quantity of any other weight, may be readily known, as in the following problems.

PROBLEM

Of irregular Solids.

P R O B L E M I.

To find the magnitude of any body, from its weight being given.

R U L E.

As the tabular specific gravity of the body
 Is to its weight in avoirdupois ounces,
 So is one cubic foot, or 1728 cubic inches,
 To its content in feet, or inches, respectively.

E X A M P L E S.

I. Required the content of an irregular block of common stone, which weighs 1 cwt., or 112 lbs.

112 lbs.

16

672

112

As (per table) 2520 is to 1792, so is 1728

1728

14336

3584

12544

1792

252,0) 309657,6 (1228, $\frac{2016}{2520} = \frac{4}{5}$ cubic inches the answer.

252

576

504

725

504

2217

2016

201,6

EXAM.

Of irregular Solids.

15

EXAM. II. How many cubic inches of gunpowder are there in one pound weight? — *Ans.* 30 nearly.

EXAM. III. How many cubic feet are there in a ton of dry oak? — *Ans.* $38 \frac{1}{2}$ cubic feet.

P R O B L E M II.

To find the weight of a body from its magnitude being given.

R U L E.

As one cubic foot, or 1728 cubic inches,
Is to the content of the body,
So is its tabular specific gravity
To the weight of the body.

E X A M P L E S.

I. Required the weight of a block of marble, whose length is 63 feet, and its breadth and thickness each 12 feet; these being the dimensions of one of the stones in the walls of Balbec.

$$\begin{array}{r} 63 \\ 12 \\ \hline 756 \\ 12 \\ \hline \end{array}$$

$$\text{As } 1 : 9072 :: 2700$$

$$\begin{array}{r} 6350400 \\ 18144 \\ \hline \end{array}$$

$$\begin{array}{l|l} \left. \begin{array}{l} 4 \\ 4 \\ 112 \\ 20 \end{array} \right\} & \begin{array}{l} 24494400 \\ 6123600 \\ 1530900 \\ 13668 \end{array} \end{array} \begin{array}{l} \text{oz.} \\ \text{lbs.} \\ \text{cwt.} \\ \text{tons.} \end{array}$$

Ans. $68 \frac{2}{3}$ tons, which is equal to the burthen of an East-India ship.

EXAM.

EXAM. II. What is the weight of a pint of gunpowder, ale measure? —
Ans. 19oz. nearly.

EXAM. III. What is the weight of a block of dry oak, which measures
 10 feet in length, 3 feet in breadth, and $2\frac{1}{4}$ deep? — *Ans.* 4335 $\frac{1}{8}$ lbs.

P R O B L E M III.

To find the specific gravity of a body.

R U L E.

CASE I. When the body is heavier than water, weigh it both in water
 and out of water, and take the difference, which will be the weight lost in
 water. Then,

As the weight lost in water
 Is to the whole weight,
 So is the specific gravity of water
 To the specific gravity of the body.

E X A M P L E.

A piece of stone weighed in air 10 lbs., but in water only $6\frac{3}{4}$ lbs. —
 Required its specific gravity.

$$\begin{array}{r}
 10 \\
 6\frac{3}{4} \\
 \hline
 \text{As } 3\frac{1}{4} : 10 :: 1000 \\
 \text{or } 13 : 40 :: 1000 \\
 \hline
 40 \\
 13 \overline{) 40000} \quad (3077 \text{ Answer} \\
 \underline{39} \\
 \cdot 100 \\
 \underline{91} \\
 \cdot\cdot 90 \\
 \hline
 \end{array}$$

CASE

CASE II. When the body is lighter than water, so that it will not quite sink, affix to it another body heavier than water, so that the mass compounded of the two may sink together. Weigh the heavier body and the mass separately, both in water and out of it, and find how much each loses in water, by subtracting its weight in water from its weight in air. Then,

As the difference of these remainders
Is to the weight of the light body in air,
So is the specific gravity of water
To the specific gravity of the body.

EXAMPLE.

Suppose a piece of elm weighs in air 15 lb., and that a piece of copper which weighs 18 lb. in air and 16 lb. in water, is affixed to it, and that the compound weighs 6 lb. in water. Required the specific gravity of the elm.

Copper.		Compound.	
18	in air.	33	in air.
16	in water.	6	in water.
Lofs	2	27	lofs.
		2	
		25	

$$\text{As } 25 : 15 :: 1000$$

$$\begin{array}{r} 15 \\ \hline 25 \overline{) 15000} \quad (600 \text{ Ans.} \\ 150 \\ \hline \dots \end{array}$$

C

PROBLEM

Of irregular Solids.

PROBLEM IV.

To find the quantities of two ingredients in a given Compound.

RULE.

Take the three differences of every pair of the three specific gravities, namely, the specific gravities of the compound, and each ingredient, and multiply the difference of every two specific gravities by the third. Then,

As the greatest product
Is to the whole weight of the compound,
So is each of the other products
To the two weights of the two ingredients.

EXAMPLE.

A composition of 112 lbs. being made of tin and copper, whose specific gravity is found to be 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and of copper 9000.

9000	9000	8784
7320	8784	7320
1680	216	1464 <i>diff.</i>
8784	7320	9000
702720	4320	13176000
52704	648	
8784	1512	
14757120	1581120	
	112	
14757120	112	13176000
	112	100 <i>copper.</i>
	26352000	12 <i>tin.</i>
	13176	
	13176	

$$14757120 : 112 :: 13176000 : 100$$

Ans. 100 lb. of copper, } in the composition.
and 12 lb. of tin,

End of the third Book.

BOOK IV.

Containing various Sorts of Prisms.

DEFINITION.

A PRISM is a solid body, whose ends are any plane figures which are equal and similar.

REMARKS.

- I. The sides of all prisms are parallelograms; but the ends or bases are of various forms: hence it is that prisms receive different names, and are accordingly called triangular, quadrangular, pentangular, hexangular, &c. — *See plates 19, 20, 21, 22, and 23.*
- II. When a prism is terminated at its ends by parallelograms, it is then most generally termed a parallelopipedon.
- III. The method of folding together the schemes contained in this book is so obvious, that nothing more need here be observed concerning it, than only to bring the contiguous parts of each figure around that particular one which is shaded.

PROBLEM.

To measure any sort of prism.

GENERAL RULE.

Measure from the center of its base or end to the middle of any one of its sides; multiply that length by half the sum of all the sides by which the prism is bounded, and that product is the area of the base; which being multiplied by the whole length of the prism, gives the solid content.

Or thus:

Square the side of the base, and multiply it by the affixed number in the following table belonging to the figure which the base of the prism is of,
C. 2 and

Of Prisms.

and that product gives the area of the base; which being multiplied by the length of the prism gives the content as before.

A GENERAL TABLE

For finding the areas of regular polygons.

Sides.	Names.	Multipliers.
3	Trigon, or equil. Δ .	,433013
4	Tetragon, or square.	1,000000
5	Pentagon.	1,720477
6	Hexagon.	2,598076
7	Heptagon.	3,633912
8	Octagon.	4,828427
9	Nonagon.	6,181824
10	Decagon.	7,694209
11	Undecagon.	9,365640
12	Duodecagon.	11,196152

EXAMPLES.

I. What is the content of an equilateral triangular prism, whose length is 12 feet, and each side $2\frac{1}{2}$ feet?

Here $2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}$ *square of the side of base.*

Then $\begin{array}{r} ,433013 \\ 6\frac{1}{4} \end{array}$

$\begin{array}{r} 2,598078 \\ 108253 \end{array}$

$\begin{array}{r} 2,706331 \text{ area of end.} \\ 12 \text{ length.} \end{array}$

32,475972 *Answer.*

EXAM.

EXAM. II. What is the content of a parallelopipedon, whose length is 6 feet, breadth 2,5 feet, and altitude 1,75 feet?—*Anf.* 26,250.

N. B. The parallelopipedon is best measured by multiplying its length, breadth, and depth, into each other.

EXAM. III. What is the content of a hexagonal prism, the length being 8 feet, and each side of its end 1 foot 6 inches?—*Anf.* 46,765368.

End of the fourth Book.

BOOK V.

Containing various Sorts of Pyramids, and Frustrums thereof.

DEFINITIONS.

I. **A PYRAMID** is a solid, having a polygon for its base, and comprehended under triangles, all meeting together in one point ; which point is called its vertex.

II. The frustrum of a pyramid is that part which remains when the top part is cut away by a plane or section passing through all its sides parallel to its base.

To fold any of the figures contained in this book.

The parts which are shaded are such evident indications, as render farther directions superfluous.

PROBLEM I.

To measure a pyramid.

RULE.

Multiply the area of its base by a third part of its perpendicular altitude ; that product is the solid content.

N. B. The perpendicular altitude is measured by a line drawn from the vertex to the center of its base.

EXAMPLES.

I. What is the solid content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet ?

$$\begin{array}{r} 1,720477 \text{ tab. area.} \\ 4 \text{ sq. side.} \\ \hline \end{array}$$

$$\begin{array}{r} 6,881908 \text{ area base.} \\ 4 \cdot \frac{1}{3} \text{ of height.} \\ \hline \end{array}$$

$$\begin{array}{r} 27,527632 \text{ Answer.} \\ \hline \end{array}$$

EXAM.

EXAM. II. Required the solidity of an hexagonal pyramid, its height being 60 feet, and each side of its base 40 feet.—*Ans.* 83138,432.

P R O B L E M II.

To measure the frustum of a pyramid.

R U L E.

Multiply the areas of the bottom and top bases together, extract the square root of that product, and add thereto the sum of both areas; that total, multiplied by one third of the frustum's altitude, gives the solidity required.

E X A M P L E S.

I. Required the content of an hexagonal frustum, whose height is 9 feet, each side of its base 4 feet, and each side of the less end 3 feet.

$$2,598076 \text{ (tab. mult.)} \times 3^2 = 2,598076 \times 9 = 23,382684 = \text{area of the less end.}$$

$$2,59807 \text{ (tab. mult.)} \times 4^2 = 2,598076 \times 16 = 41,569216 = \text{area of the greater end.}$$

$$\sqrt{41,569216 \times 23,382684} = \sqrt{971,999841855744} = 31,176911.$$

$$41,569216 + 23,382684 + 31,176911 \times \frac{9}{3} = 96,128811 \times 3 = 288,386433 \text{ feet} = \text{solidity required.}$$

EXAM. II. What is the solidity of the frustum of a pentagonal pyramid, the side of whose greater end is 1 foot 6 inches, that of the lesser end 6 inches, and the length 5 feet?—*Ans.* 9,319250 feet.

End of the fifth Book.

B O O K VI.

*Containing an Illustration of some Theorems in EUCLID'S
eleventh and twelfth Books.*

EUCL. XI. Prop. XXVIII.

T H E O R E M.

A PLANE passing through the diameters of opposite planes of a parallelopipedon divides it into two equal prisms. See this illustrated in Plates XXXIV. and XXXV.

Directions for folding the schemes contrived for illustrating the above theorem.

P L A T E XXXIV.

I. Raise up one half of the figure, and fold it so that the point E may fall upon A, and the point F upon B, and raise up the parallelogram at the end; thus you have one of the prisms.

II. Then bring over the other part of the scheme, so that the corner C may likewise fall upon A, and the corner D upon B; thus will there be formed the whole parallelopipedon, and its section made by the plane passing through its diagonal A B.

P L A T E XXXV.

I. Lift up the figure, and bring the point A to C, and the point B to D.

II. Fold back the rest of the figure upon the line A B, which is properly cut for that purpose; so that the two parallelograms which are separated by A B may lie close together.

III. Then bring over the remaining part of the scheme, so that the points E and F may coincide with C and D; then folding together the four triangles, the whole parallelopipedon will be formed, and also its section made according to a plane passing through the diagonal of its base or end.

EUCL.

EUCL. XII. *Prop.* III.

T H E O R E M.

A TRIANGULAR pyramid may be divided into two equal triangular pyramids, having each the same base and altitude, and into two equal triangular prisms, which two prisms are together greater than the half of the whole pyramid.

Directions for folding together the scheme which explains this proposition.

P L A T E XXXVI.

I. First form the triangular pyramid, A C B, by making the line B C coincide with A C.

II. Fold back upon the line B C the triangle numbered 4, so that it may lie close to that which is marked 3.

III. Then fold over the sides 5, 6, and 7, bringing the line D E so that D may fall upon A, and E upon C, which, with the quadrilateral base that is shaded, forms one of the prisms.

IV. Then fold back the other part of the figure, so that the side 8 may be close to that marked 7; thus will there remain a triangular vacuity at the base of the pyramid A C B, which must be filled by folding together the sides marked 9 and 10, making the point F fall upon A, and the point G upon C; thus will the other prism be formed.

V. This done, there remains only to form the other pyramid G I H, which is effected by folding its parts back upon the line G K, and folding round the triangles marked 12, 13, 14; the partition lines whereof are all cut on the front side of the figure, which being folded accordingly, completes the whole pyramid, and exhibits a view of the several sections described in the above theorem.

EUCL. XII. Prop. VII.

T H E O R E M.

EVERY pyramid is the third part of a prism, having the same base and height. *See this illustrated in a triangular prism by*

P L A T E XXXVII.

To fold the figure contained in this plate.

I. First form the pyramid A B C, by bringing the point C to A, so that the line B C may fall upon A B.

II. Then form the pyramid B A D, by bringing D to B, making A D coincide with A B, and raising up the triangle at B.

III. Then turn back the rest of the figure upon the line A D, so that the triangle marked 7 may closely adhere to that marked 6.

IV. Lastly, The vacuity that is now between the two pyramids so formed will be filled exactly, by folding together the pyramid contained under the triangles marked 8, 9, 10, in such sort that E be connected with A, and F with B; thus is the whole prism completed, and the sections above described clearly seen.

N. B. * When the last-mentioned pyramid is introduced into the space remaining between the other two, by pressing a little upon that part which is the upper edge or side of the prism, the formation of the prism will be rendered more perfect.

BOOK VII.

Containing the Conic Sections.

DEFINITIONS.

I. **T**HE *Conic Sections* are such figures as are formed by the cutting of a *cone* in various directions.

II. A cone is a solid, described by the revolution of a right-angled triangle about one of its legs, which remains fixed.

III. The *axis* of the cone is that fixed right line about which the triangle revolves.

IV. The extreme end or point, by which the cone is terminated, is called its vertex.

V. The frustum of a cone is the part remaining when the top is cut off by a plane or section passing through it parallel to its base.

REMARKS.

I. The cone and its frustum are measured by the same rules as obtain for pyramids and their frustums.

II. As a pyramid is the third part of a prism, having the same base and altitude, so in like manner a cone is the third part of a cylinder, having the same base and altitude; but the roundness of those figures does not admit of an explanation of this truth by this mechanical method of representing them.

Directions for folding together the schemes contained in this book.

PLATE XXXVIII.

I. Bend the arc A B evenly round the arc E F, thereby causing A to come to E, and B to F.

II. Turn about the triangle B C D upon the side B C, so as to make B D coincide with the diameter of the base E F, and A C to coincide with D C; thus will one half of the cone be formed.

D 2

III. In

III. In like manner form the other half, by bringing G to E, and H to F, the line G K even upon E F, and the sides H I and K I even to each other; so will the whole cone, and the section through its axis, both appear.

P L A T E XXXIX.

I. Fold the arc A D round the base, so that A may come to B, and the arc D C in like manner, making C to unite A at the point B.

II. Make the lines A F unite together, which will form the frustum of a cone cut parallel to its base, and shew the section to be a circle.

P L A T E XL.

Fold the arcs A B and B C round the arcs B D E, B F E, and make the sides A G and C H coincide every where together upwards from the point E, which will represent a cone cut obliquely to its base, and shew the section to be an ellipsis.

P L A T E XLI.

I. Form the whole part of the cone A C B, by bringing the points A and B together, and bending it so as to become round as near as may be.

II. Bend regularly the other part of the figure, so that the arcs D E, F E, may surround the arcs F G, F H.

III. Raise up the parabola G I H, making the point I meet the coincident points A and B; thus may you see the section made by cutting a cone parallel to one of its sides.

P L A T E XLII.

The last directions obtain here; for bringing E and F together, and rounding the part E D E, as was there described, and the points A and B being brought to G and H, if the hyperbola G I H be then raised up, as before directed, the section of a cone, cut parallel to its axis, will be thereby presented to view:

P R O B L E M

P R O B L E M I.

To find the solidity of a cone.

R U L E.

Multiply the square of the diameter of the base by ,7854; the product being multiplied by $\frac{1}{3}$ of the height will produce the solidity.

E X A M P L E S.

I. Required the solidity of a cone, whose base is 20 feet in diameter, and its perpendicular height 24 feet.

$$\begin{array}{r}
 \text{,7854} \\
 \times 400 \text{ square diameter.} \\
 \hline
 314,1600 \\
 \times 8 \frac{1}{3} \text{ of height.} \\
 \hline
 2513,2800 \text{ solidity required.} \\
 \hline
 \end{array}$$

EXAM. II. Required the solidity of a cone, the diameter of whose base is 18 inches, and its altitude 15 feet. — *Ans.* 8,83575 feet.

P R O B L E M II.

To find the solidity of the frustrum of a cone.

R U L E.

Add together the squares of the two diameters and the product of the two; which multiply by $\frac{1}{3}$ of ,7854, or ,2618; this will give the mean area, which being again multiplied by the height of the frustrum will give the content.

EXAM.

Of the Conic Sections.

EXAMPLES.

I. What is the content of the frustrum of a cone, whose height is 20 inches, and the diameters of its two ends 28 and 20 inches?

28	28	20
28	20	20
—	—	—
224	560	400
56	784	—
—	400	
784	—	
—	1744	
	,2618	
	—	
	13952	
	1744	
	10464	
	3488	
	—	
	456,5792	
	20	
	—	
	9131,5840	Answer.
	—	

II. What is the content of the frustrum of a cone, the height being 18 inches, the greatest diameter 8, and the least 4?—*Ans.* 527,7888.

IT will not perhaps be considered improper to conclude these few pages with the methods of finding the superficial and solid contents of a globe or sphere. The solid itself is so universally known, that any figure of it here may be thought superfluous or unnecessary.

PROBLEM

PROBLEM I.

To find the convex surface of a sphere or globe.

RULE.

Multiply the diameter by 3,1416, and the product will be the circumference; or divide the circumference by 3,1416, and the quotient will be the diameter. Then multiply the diameter by the circumference, and the product will be the convex superficies required.

Note. — The curvilinear surface of any zone or segment, will also be found by multiplying its height by the whole circumference of the sphere.

EXAMPLES.

I. What is the convex superficies of a globe, whose diameter, or axis, is 24 inches?

$$\begin{array}{r}
 3,1416 \\
 24 \\
 \hline
 125664 \\
 62832 \\
 \hline
 75,3984 \text{ circumference.} \\
 24 \\
 \hline
 3015936 \\
 1507968 \\
 \hline
 1809,5616 \text{ Answer.}
 \end{array}$$

II. Required the area of the surface of the earth, its diameter, or axis, being $7959\frac{3}{4}$ miles, or its circumference 25000 miles. — *Ans.* 198943750 square miles.

III. What is the convex surface of a spherical zone, whose breadth is 4 inches, and cut from a sphere of 25 inches diameter? — *Ans.* 314,16 inches.

PROBLEM

Of the Globe, or Sphere.

P R O B L E M II.

To find the solidity of a sphere, or globe.

R U L E.

Multiply the cube of the diameter by ,5236, and the product will be the solidity.

E X A M P L E S.

I. What is the solidity of a sphere, whose axis is 12?

12	Or thus.
12	5236
—	12
144	—
12	6,2832
—	12
1728	—
,5236	75,3984
—	12
10368	—
5184	904,7808 <i>Answer.</i>
3456	—
8640	—
—	—
904,7808 <i>Answer.</i>	—

II. What is the solid content of the earth, its diameter being $7957\frac{3}{4}$ miles?
— *Ans.* 26385743776 miles.

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Plate.I.

Book.1.

The
Tetraedron

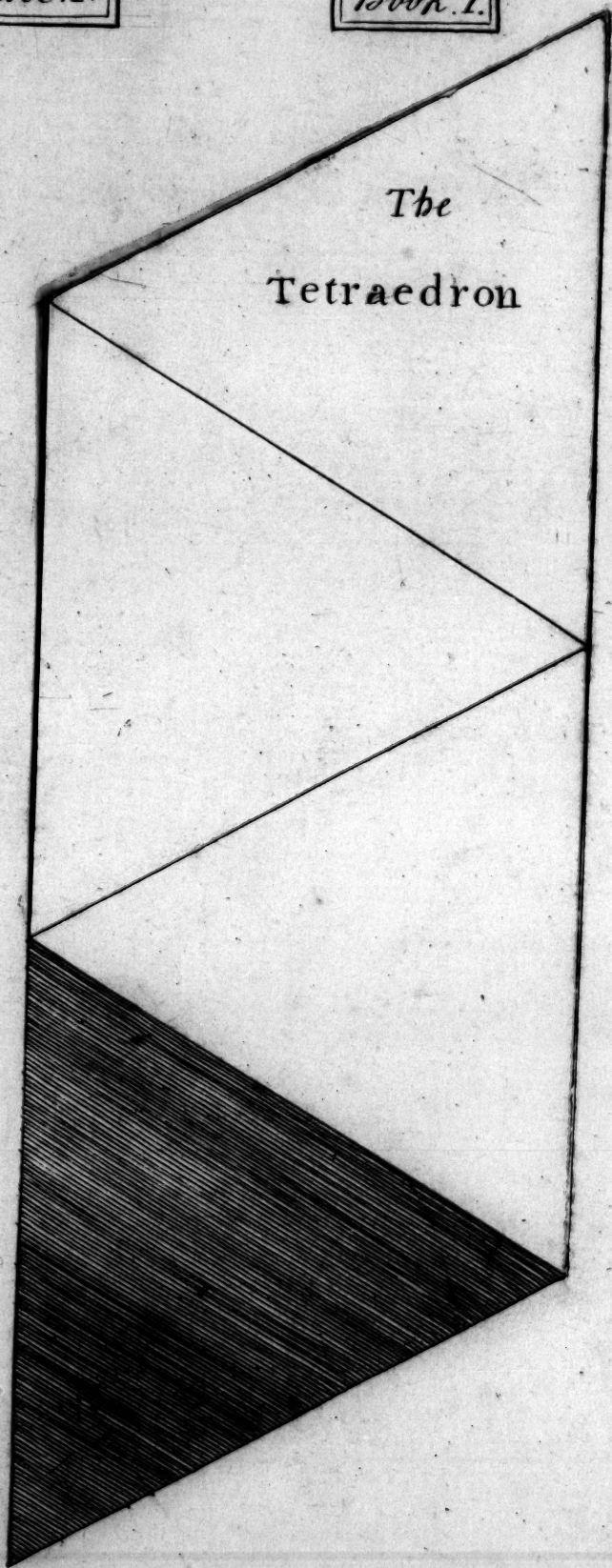
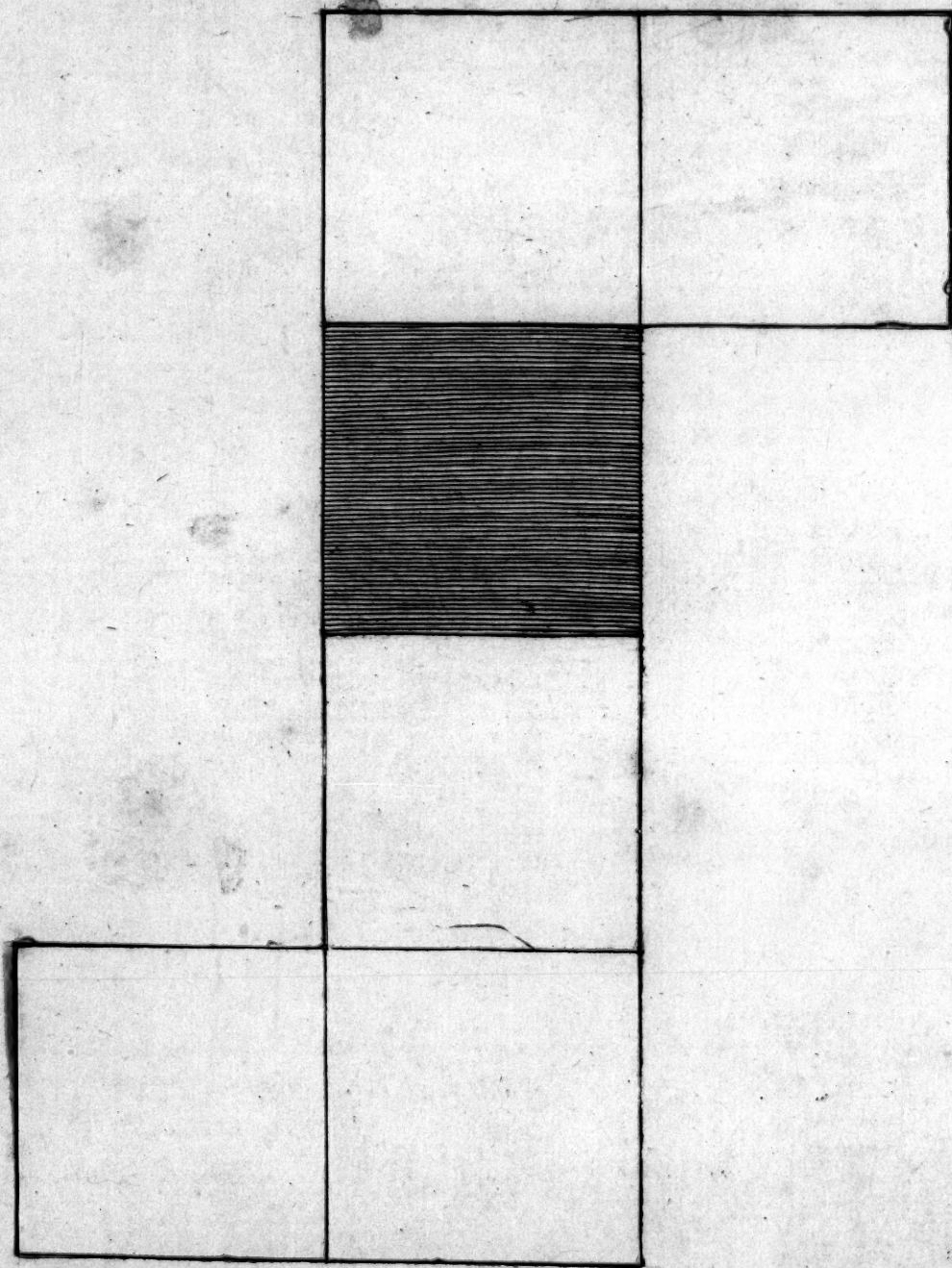


Plate. II.

The
Hexaedron or Cube.

Book. I.



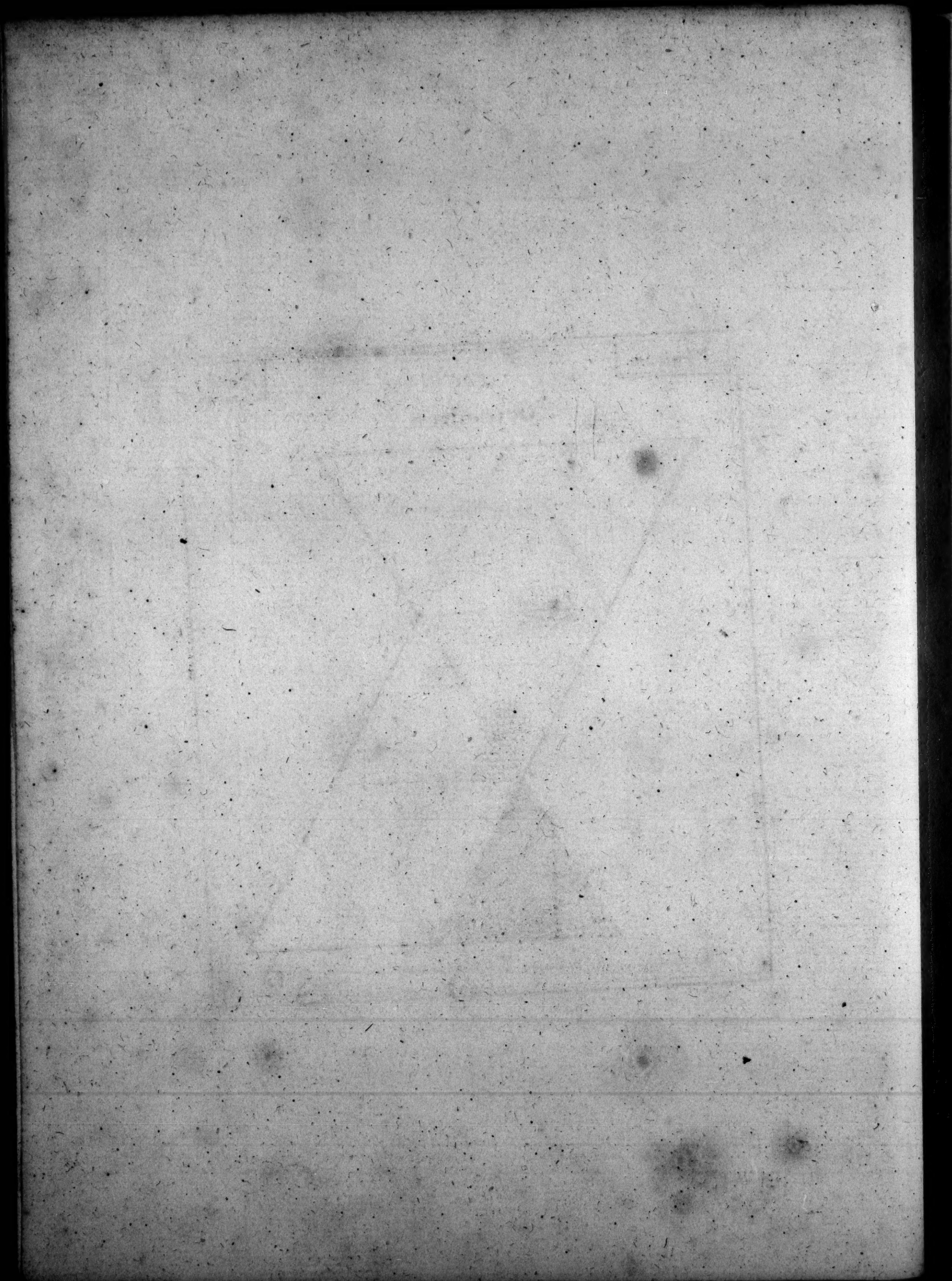


Plate. III.

The
Octoedron

Book. I.

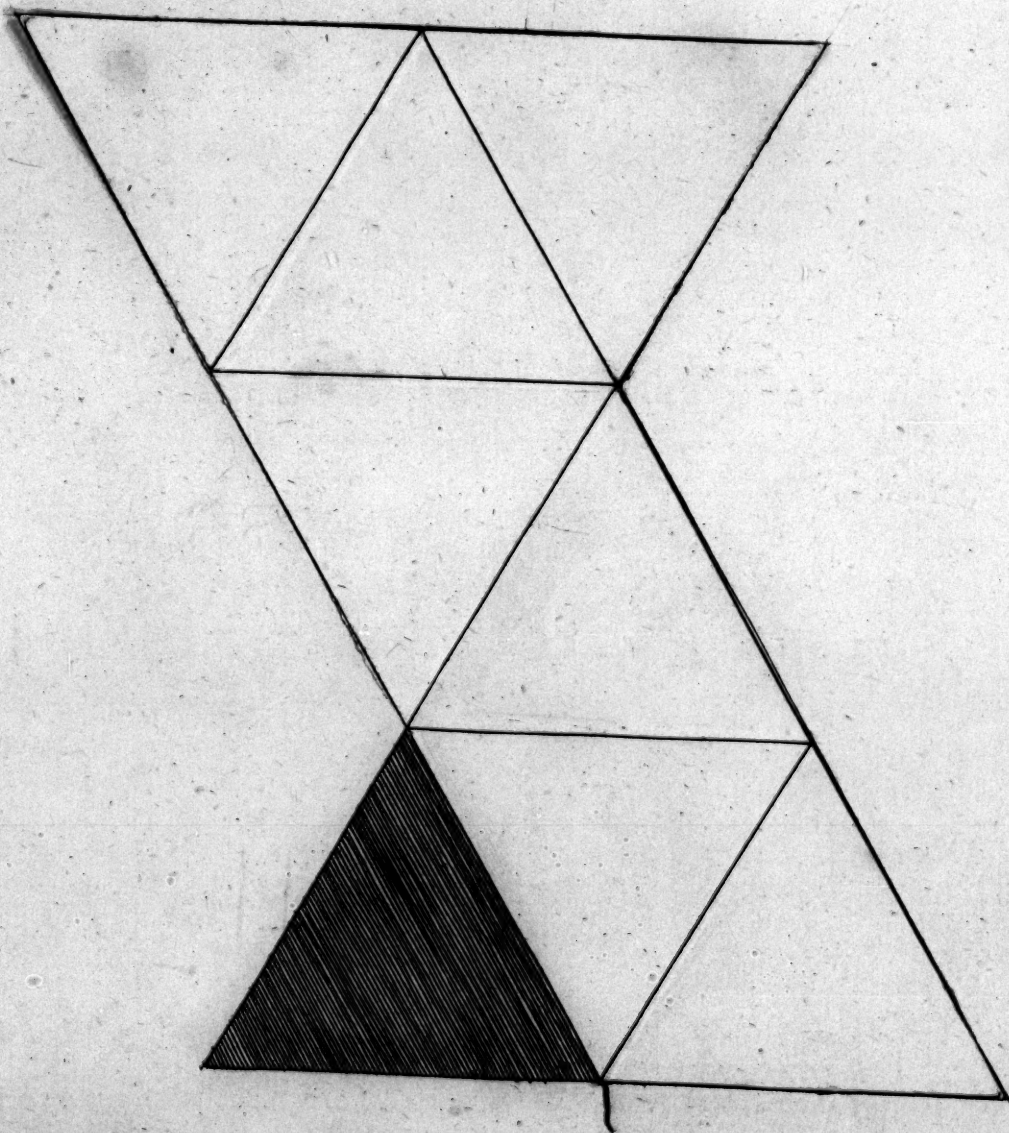
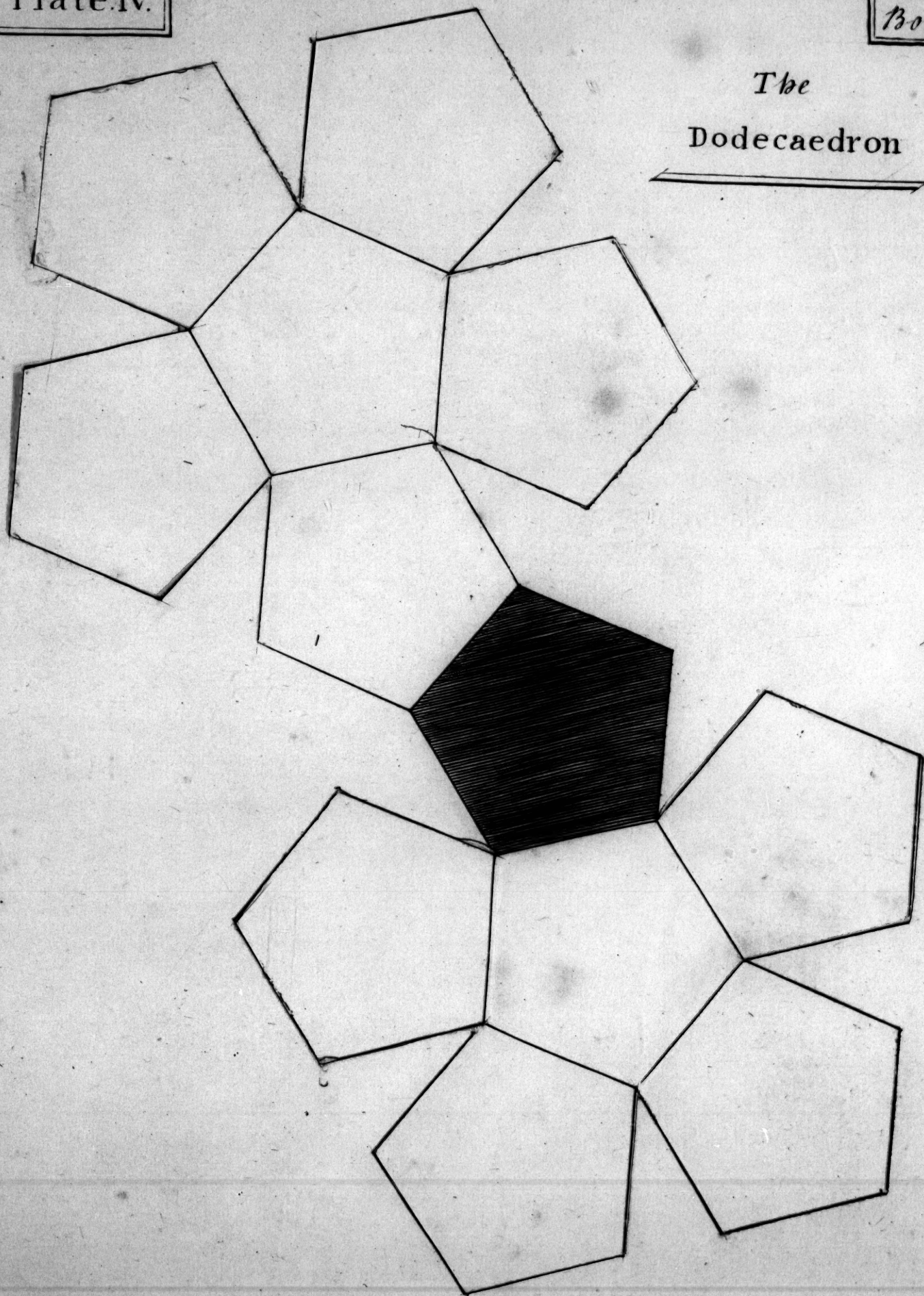


Plate.IV.

Book.1.

The
Dodecaedron



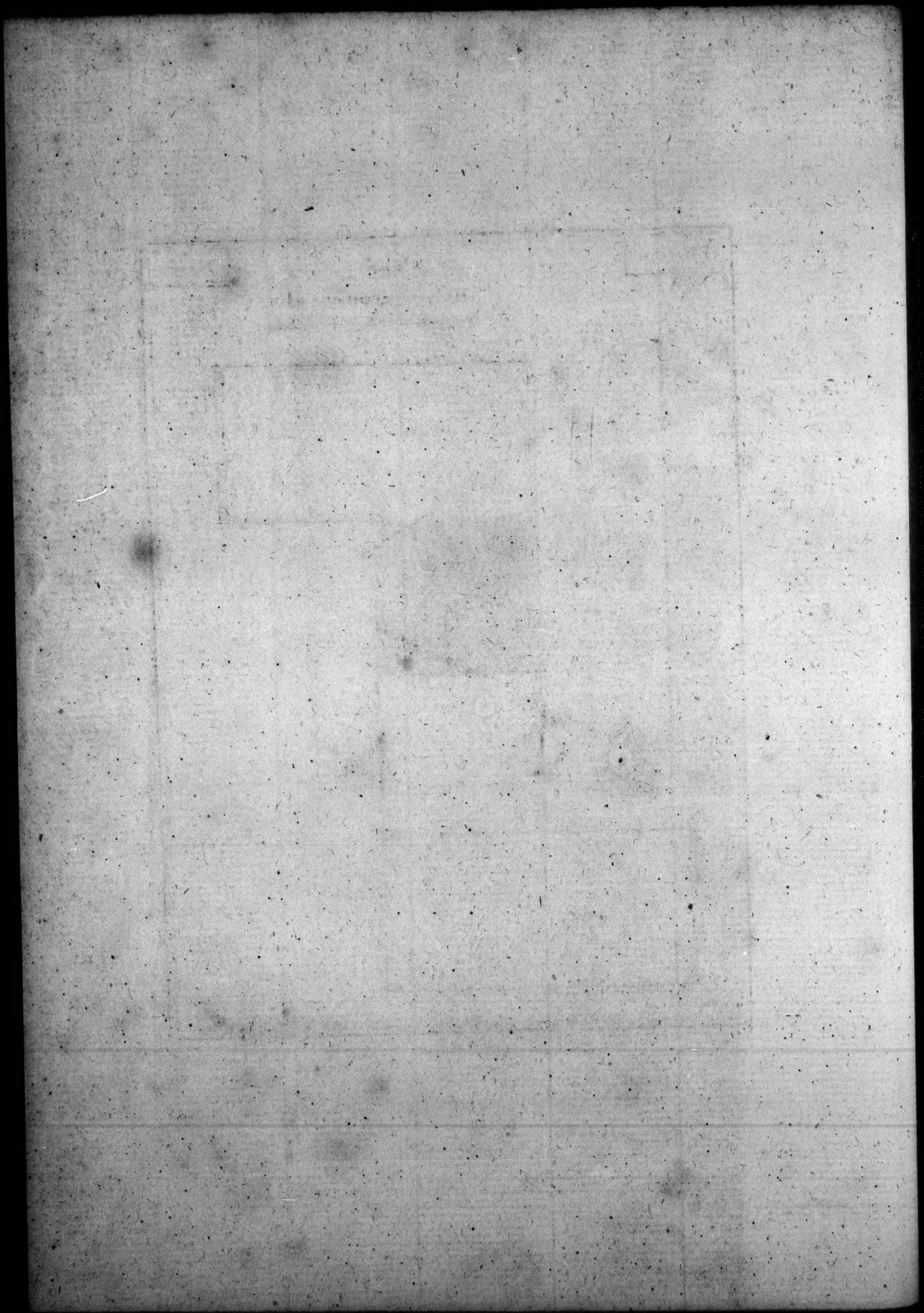
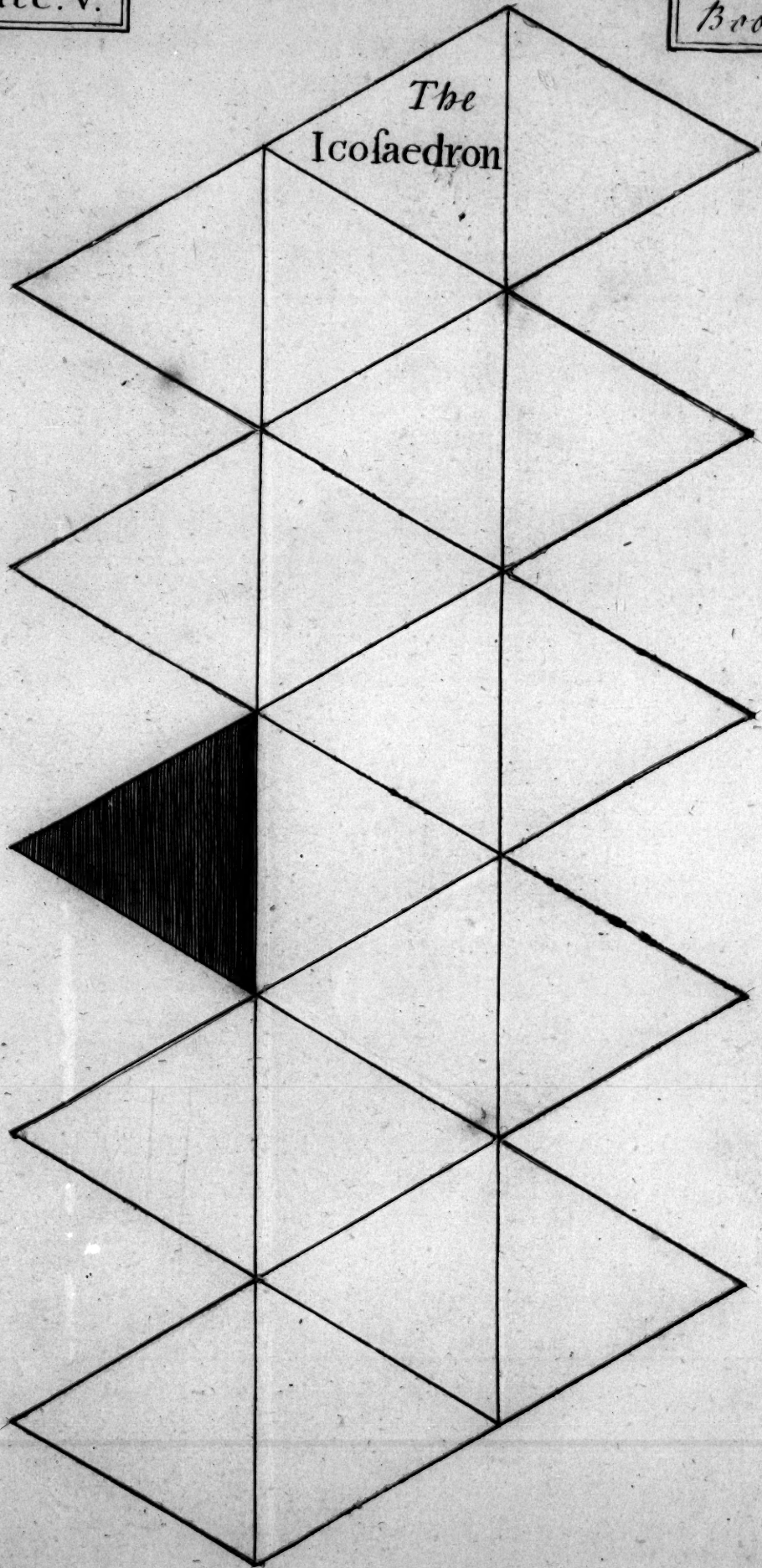


Plate.V.

Book.I.

The
Icofaedron



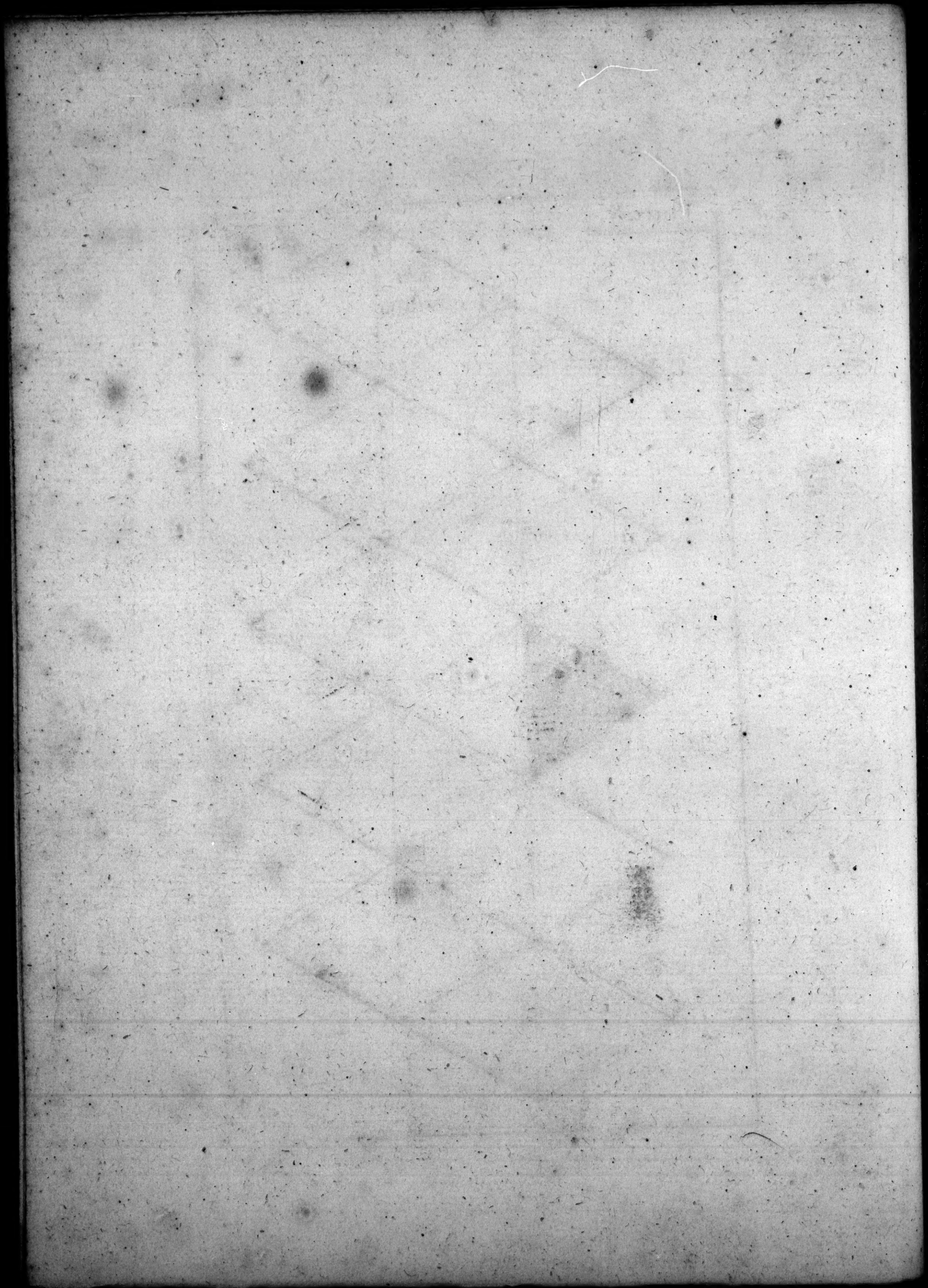
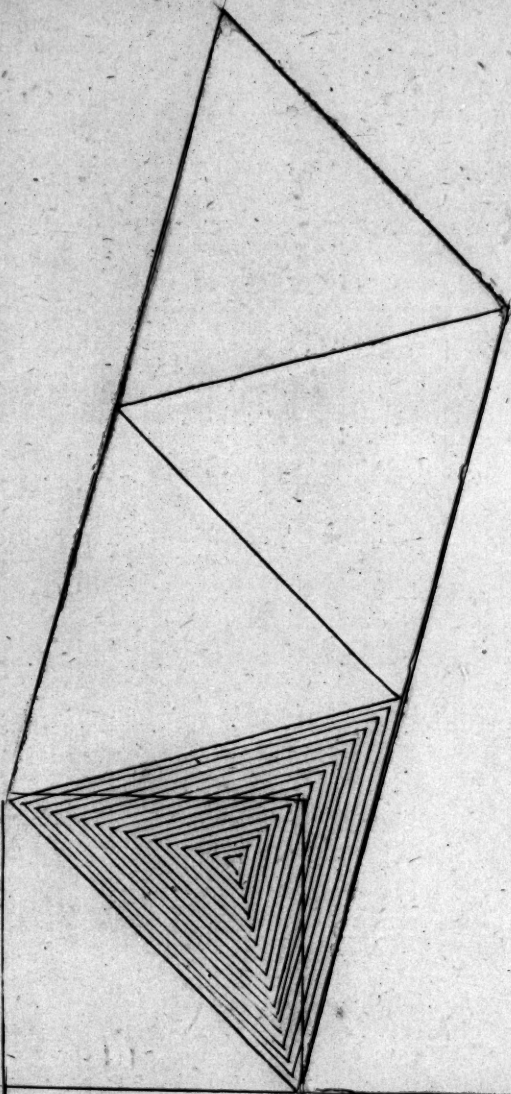


Plate VI.

Book. 2.



*The
Tetraedron.
inscribed in an
Hexaedron.*

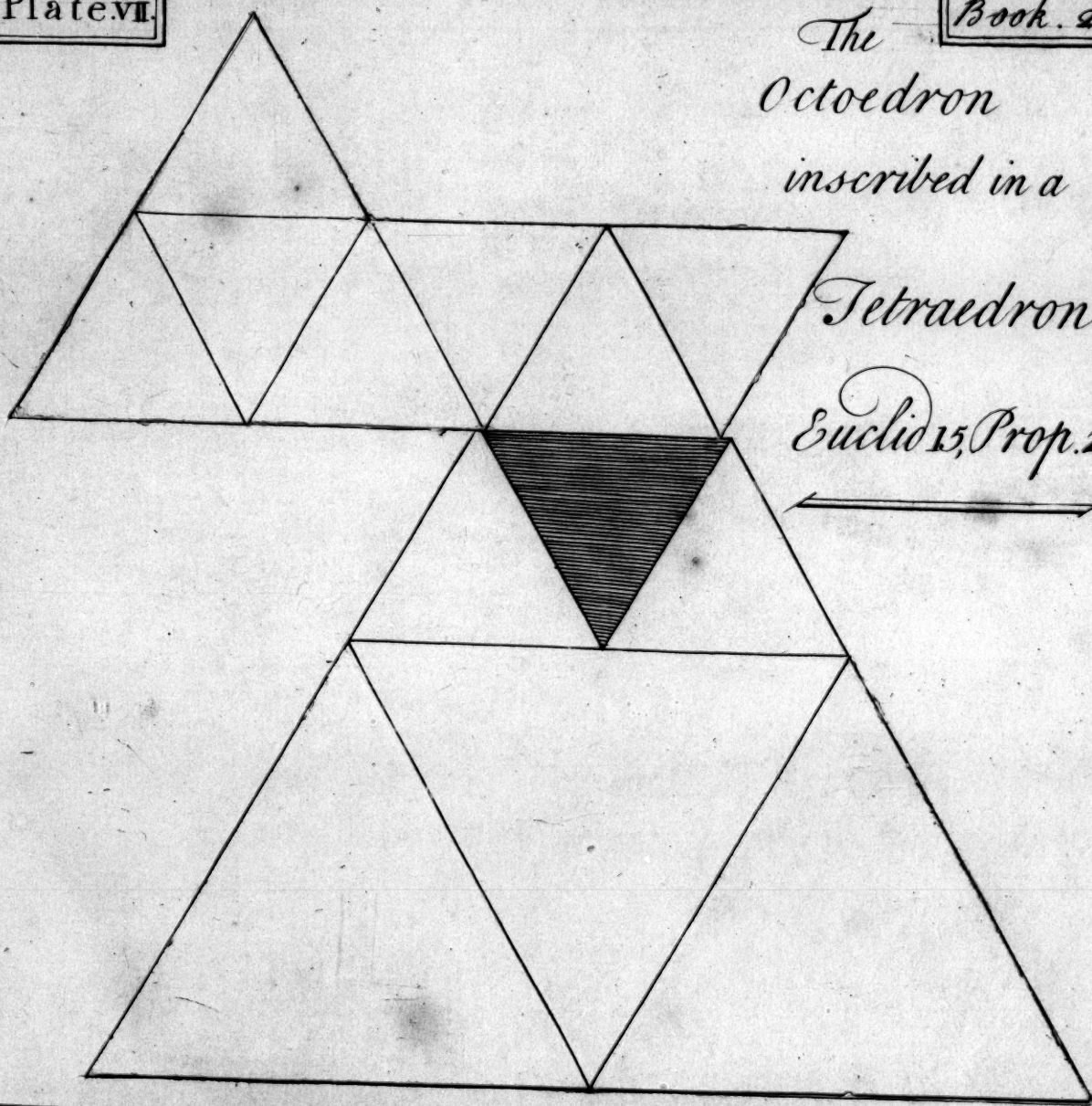
Eucldid. 15. Prop. 1.



Plate VI.

Book. 2.

The
Octoedron
inscribed in a
Tetraedron.
Euclid 13, Prop. 2.



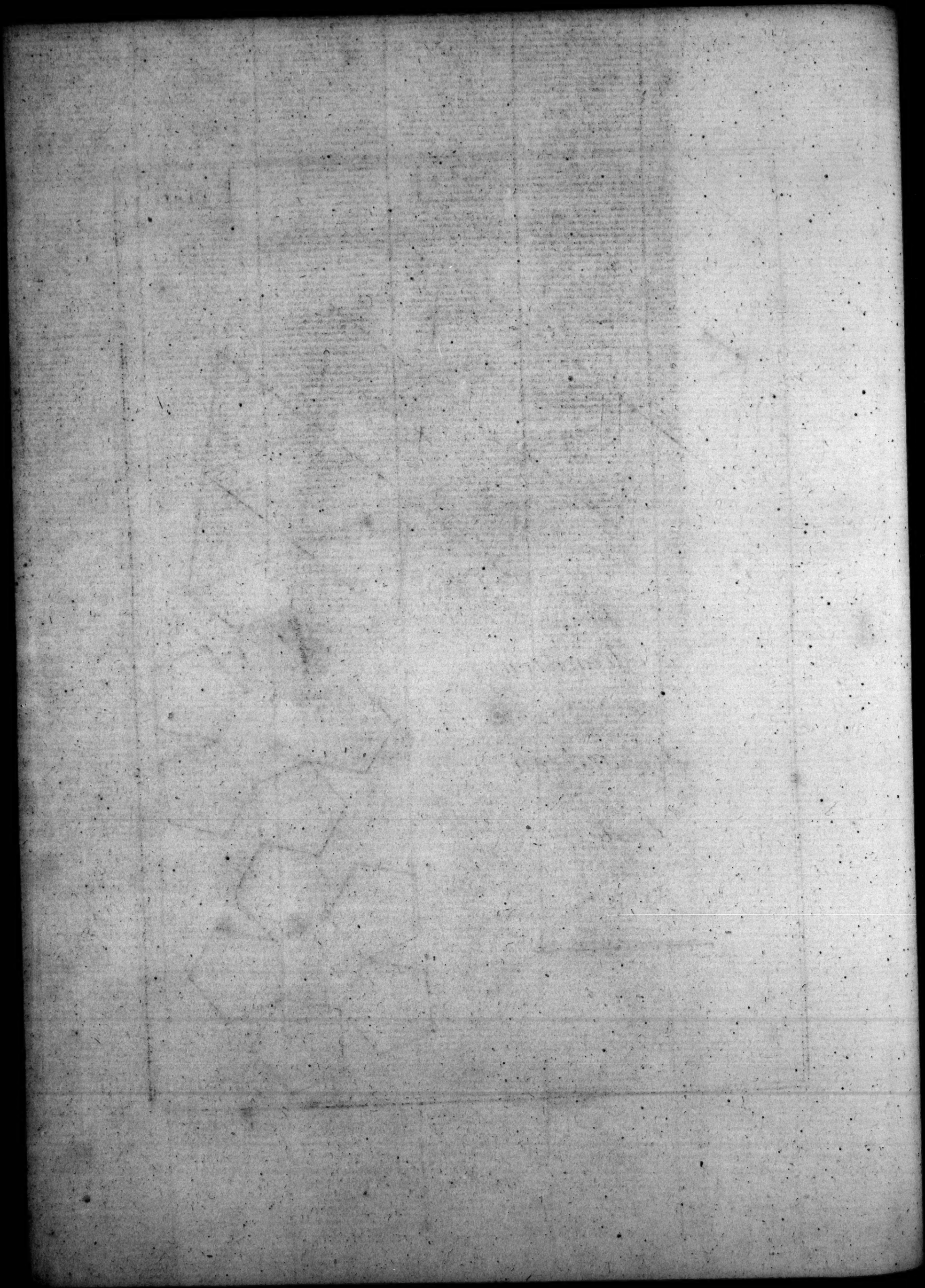


Plate. VIII.

Book. 2.

The
Octoedron
inscribed in the
Hexaedron.

Euclid 15.

Prop. 3.

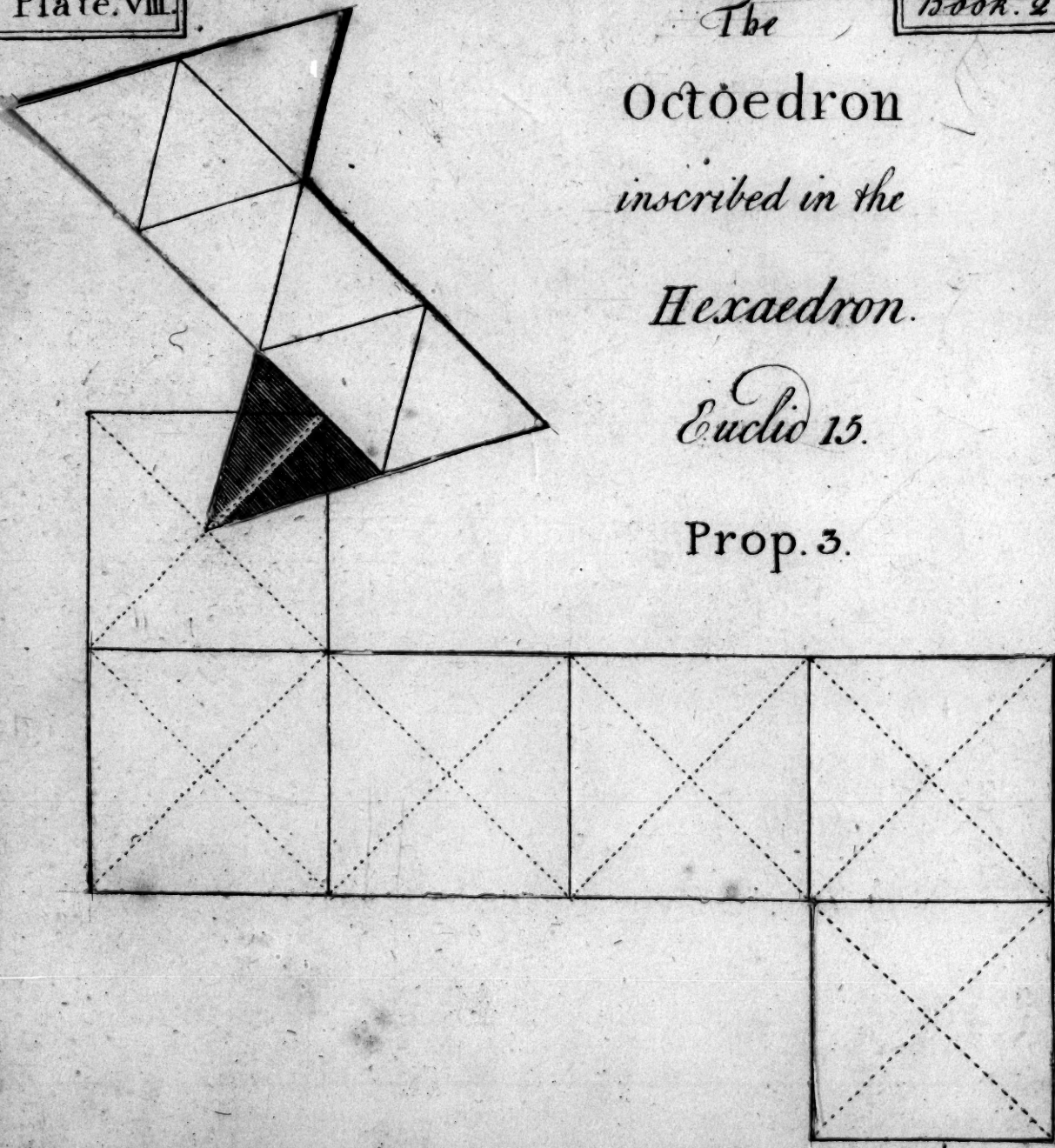
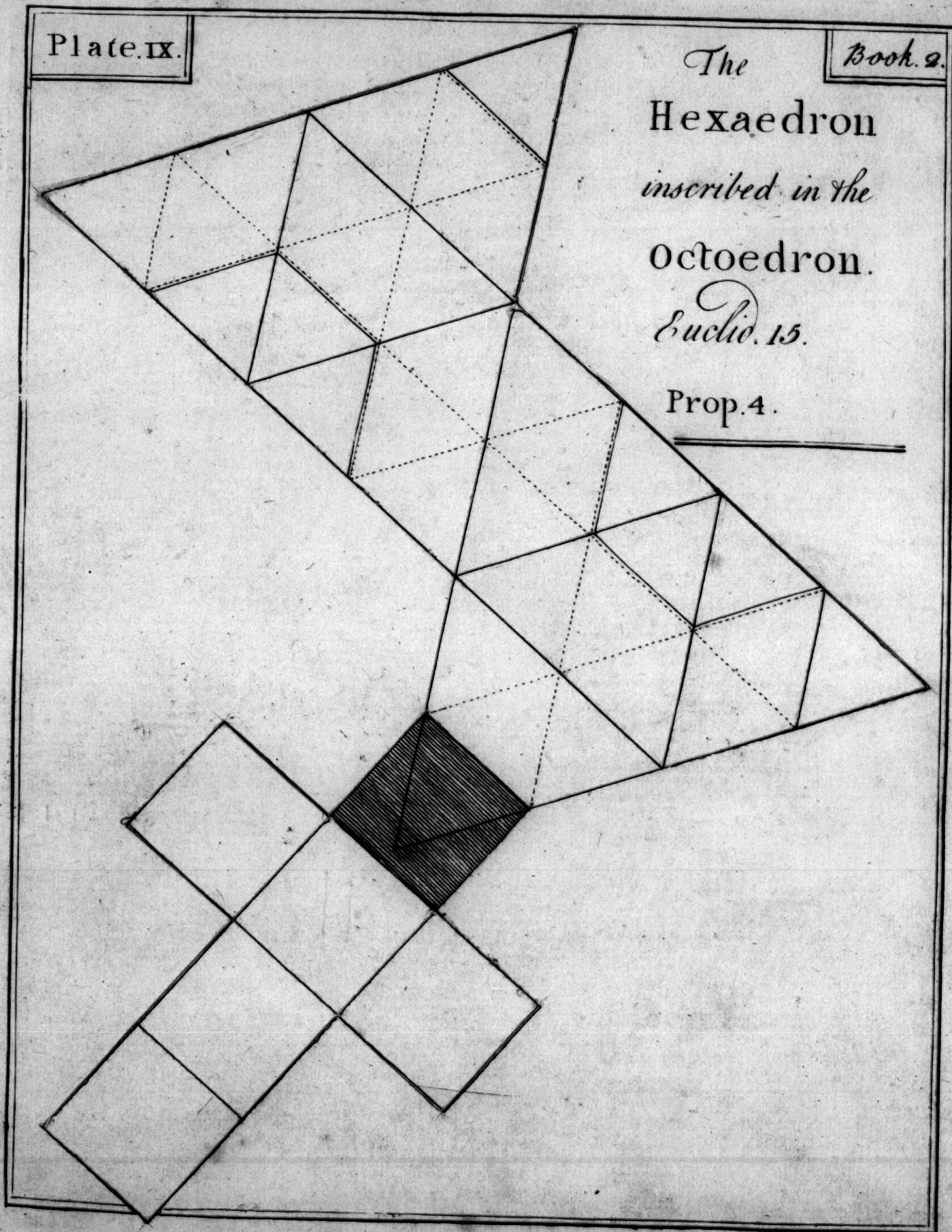


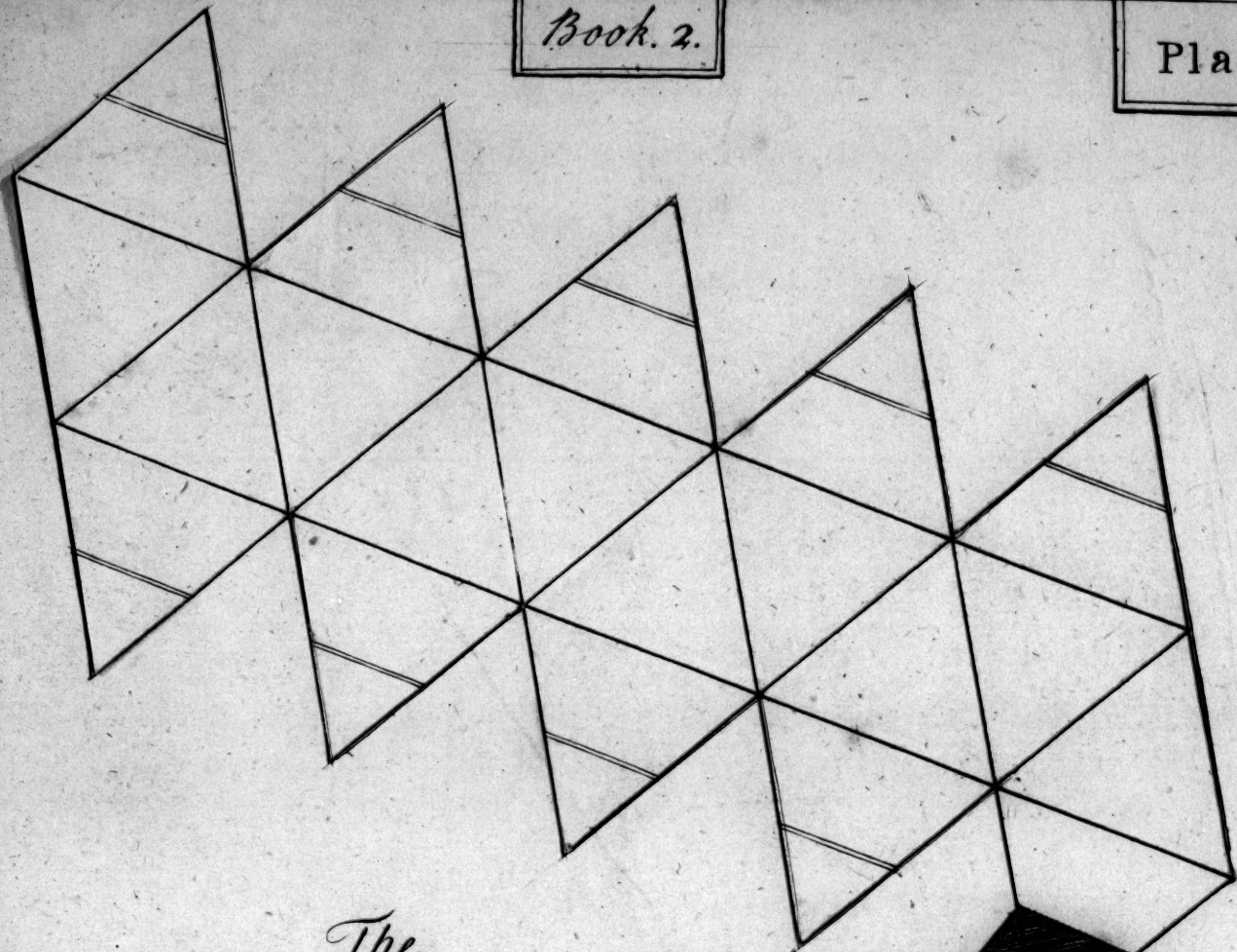
Plate.IX.

Book. 2.

The
Hexaedron
inscribed in the
Octoedron.
Euclid. 13.

Prop. 4.

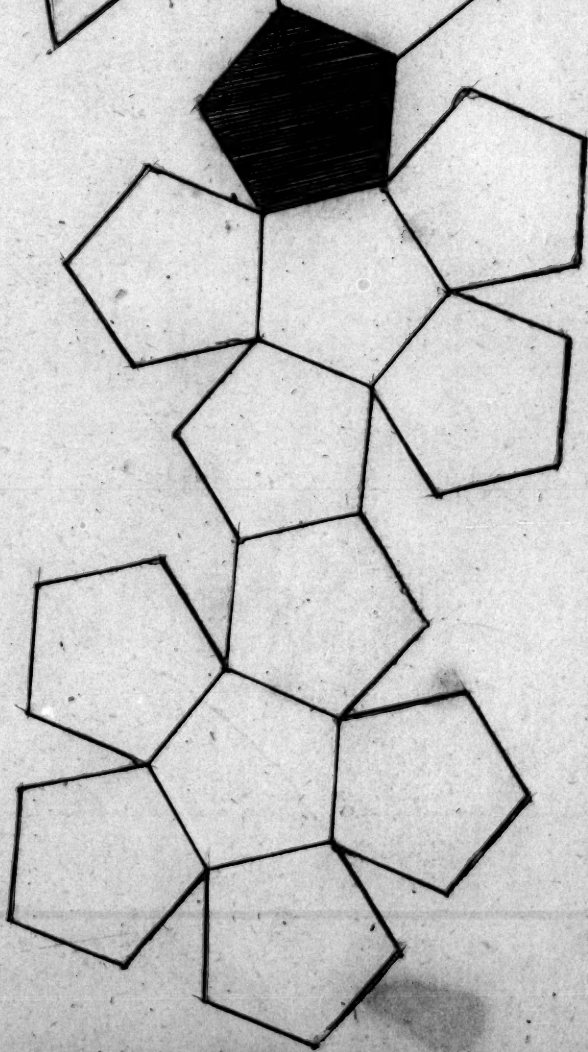




The
Dodecaedron
inscribed in an
Icosaedron.

Euclid. 13.

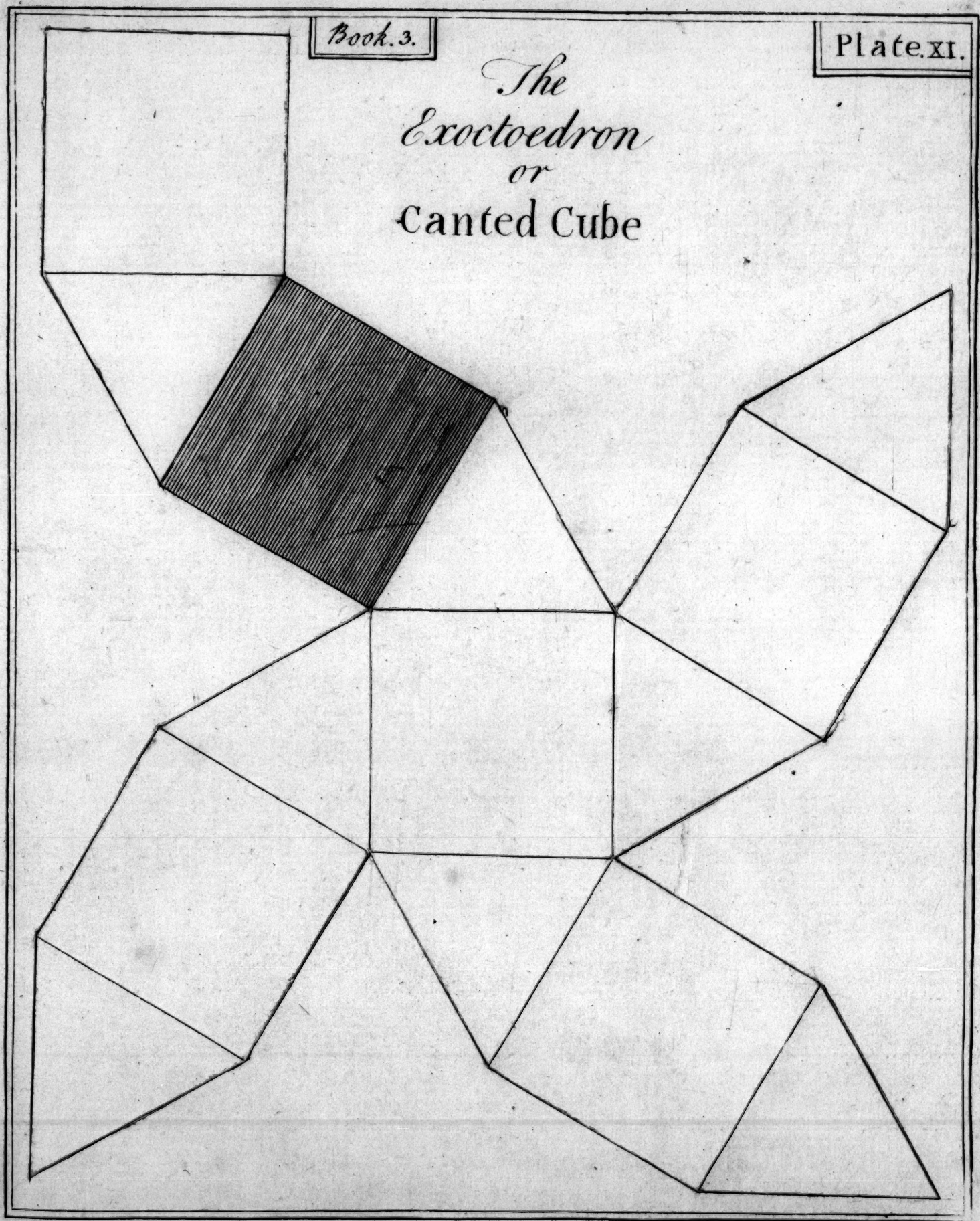
Prop. 5.



Book. 3.

Plate. XI.

The
Exoctoedron
or
Canted Cube

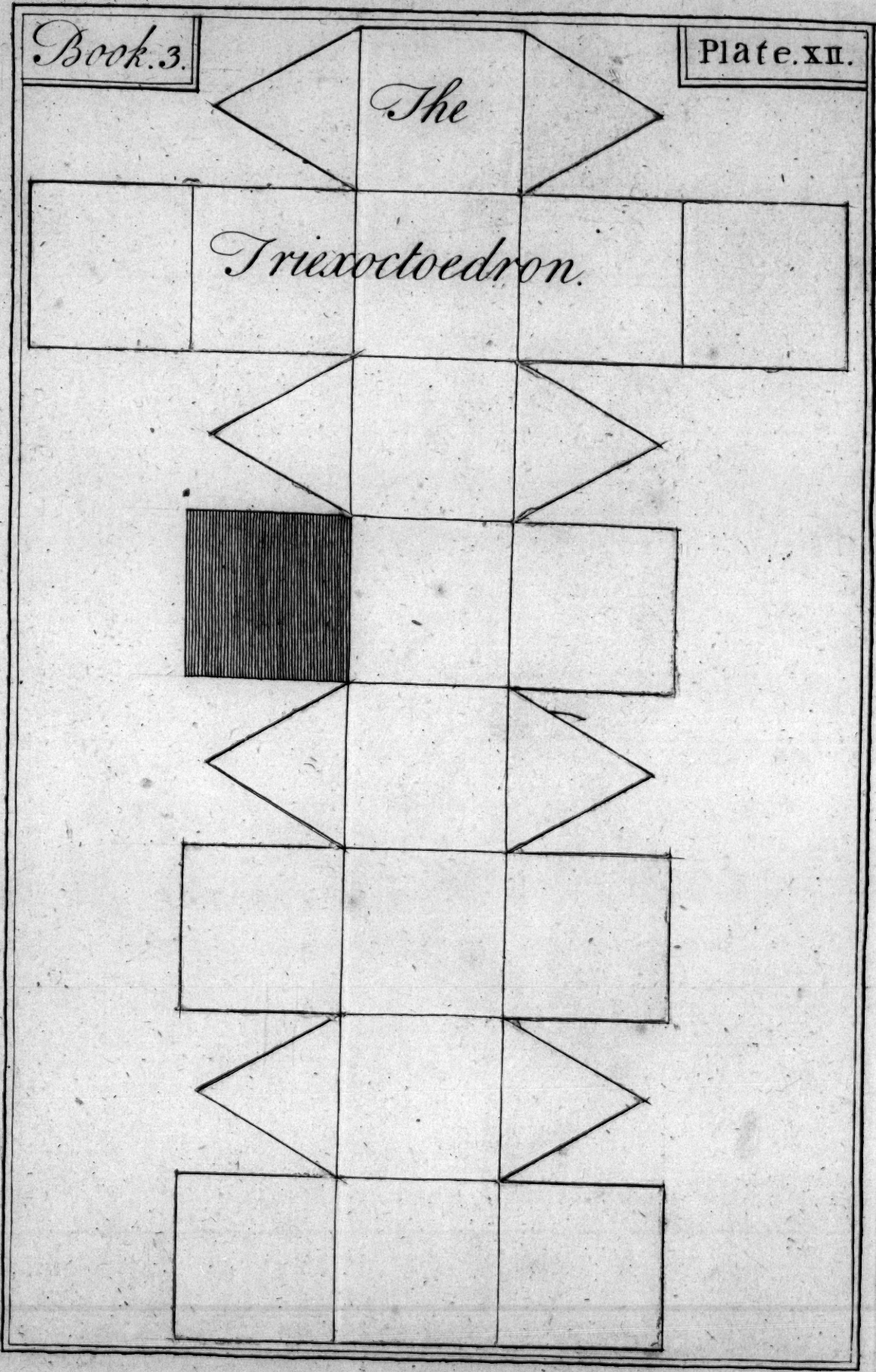


Book.3.

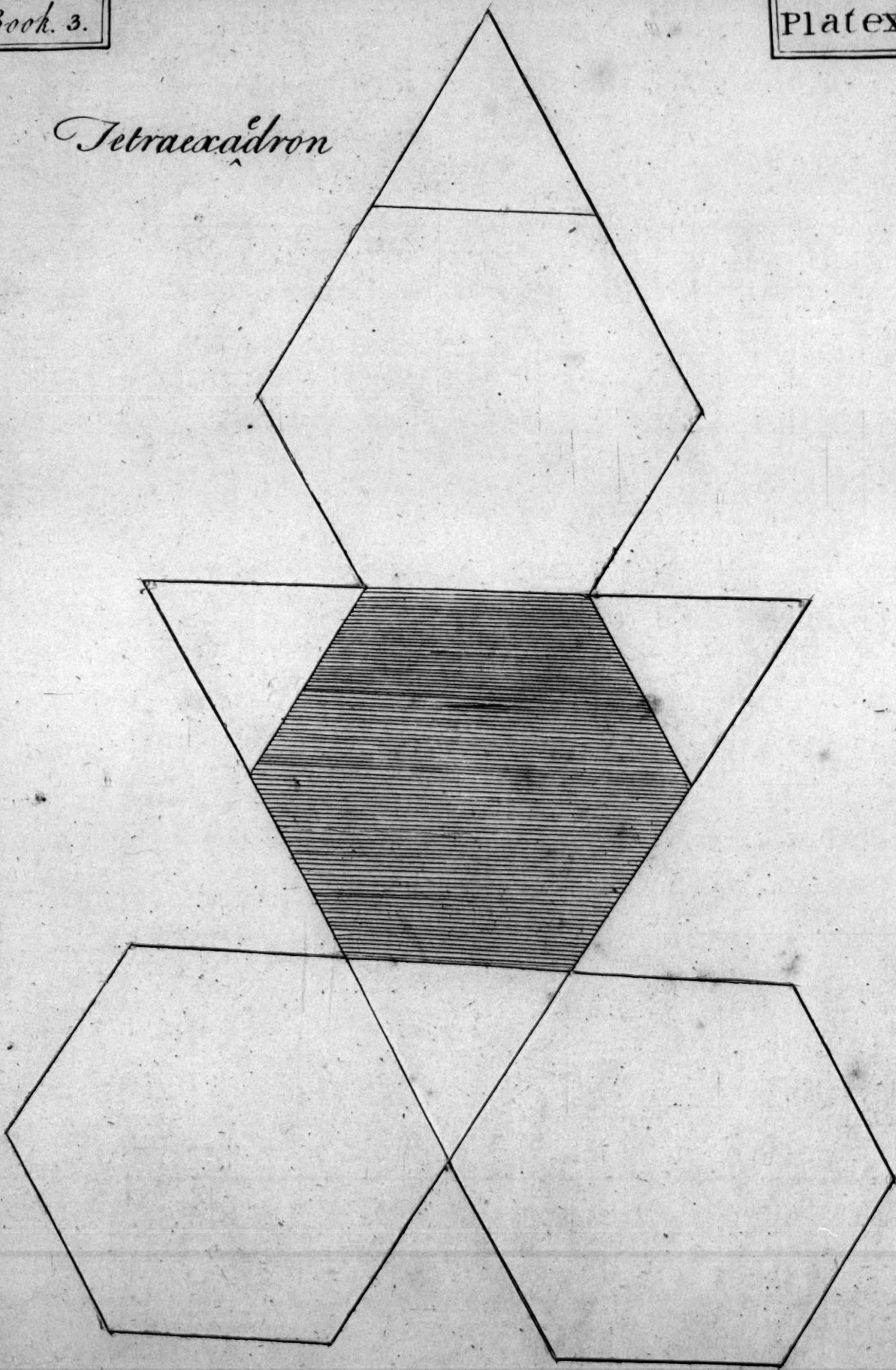
Plate.XII.

The

Triaxoctaedron.

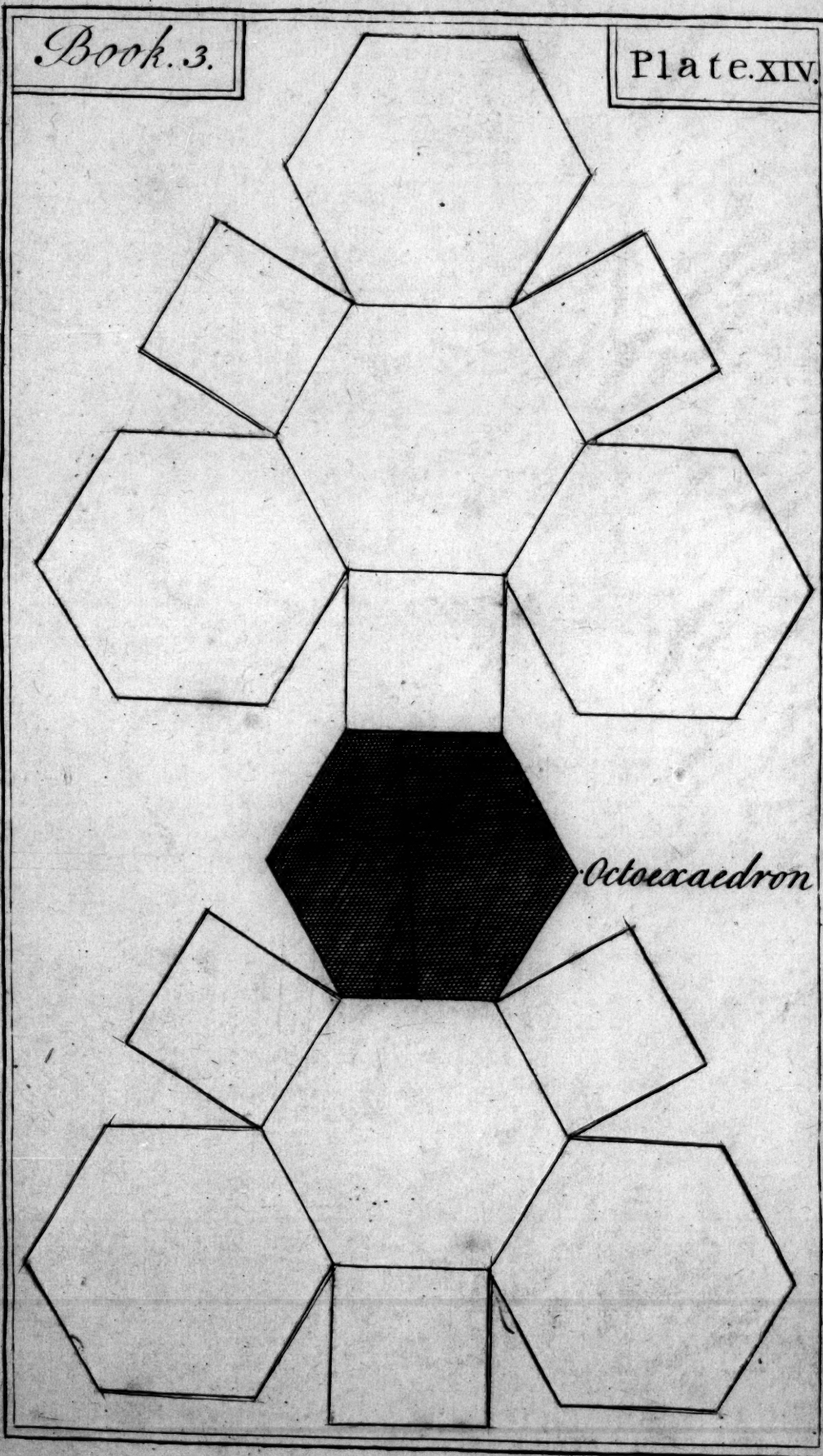


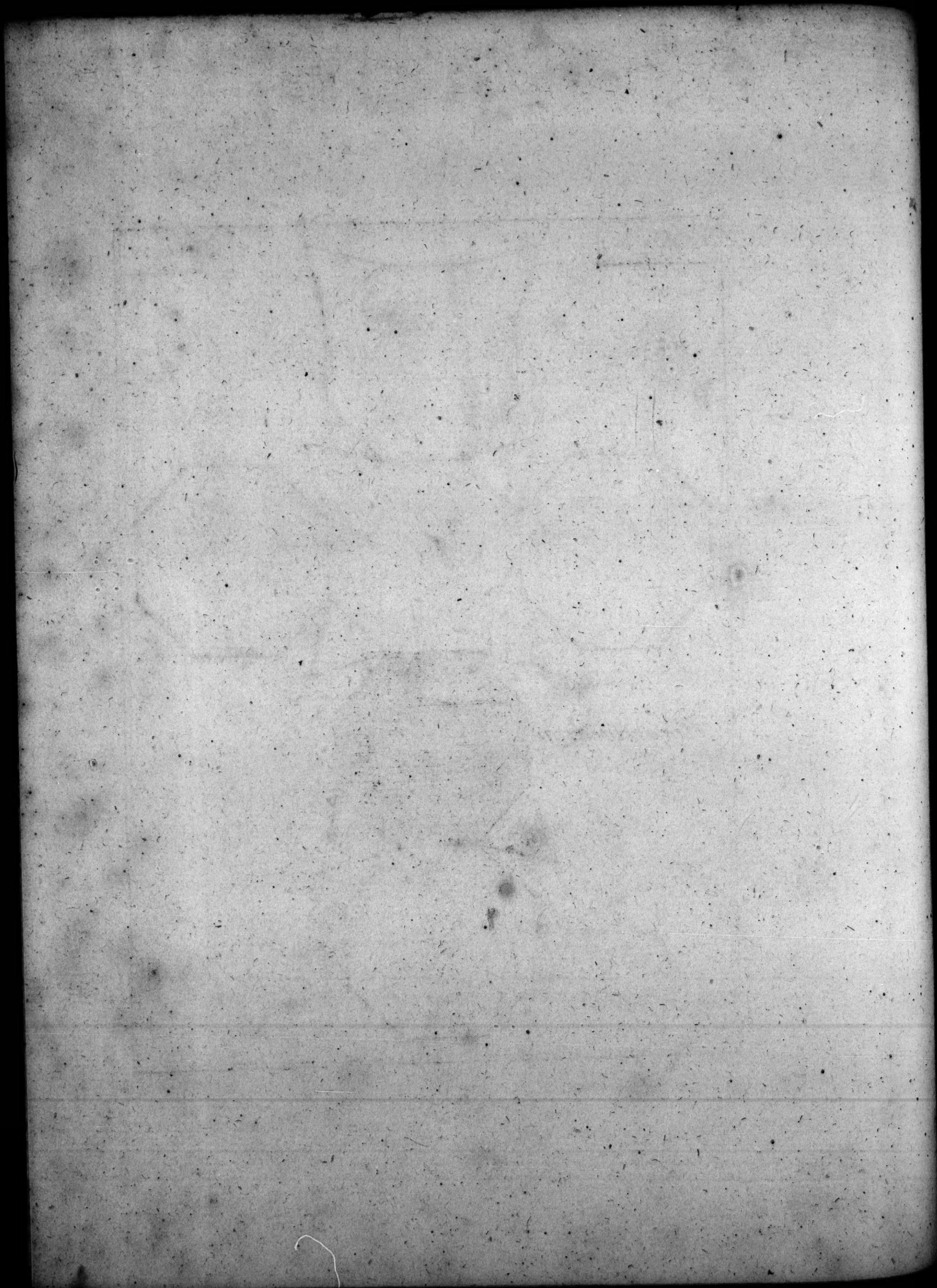
Tetraexadron

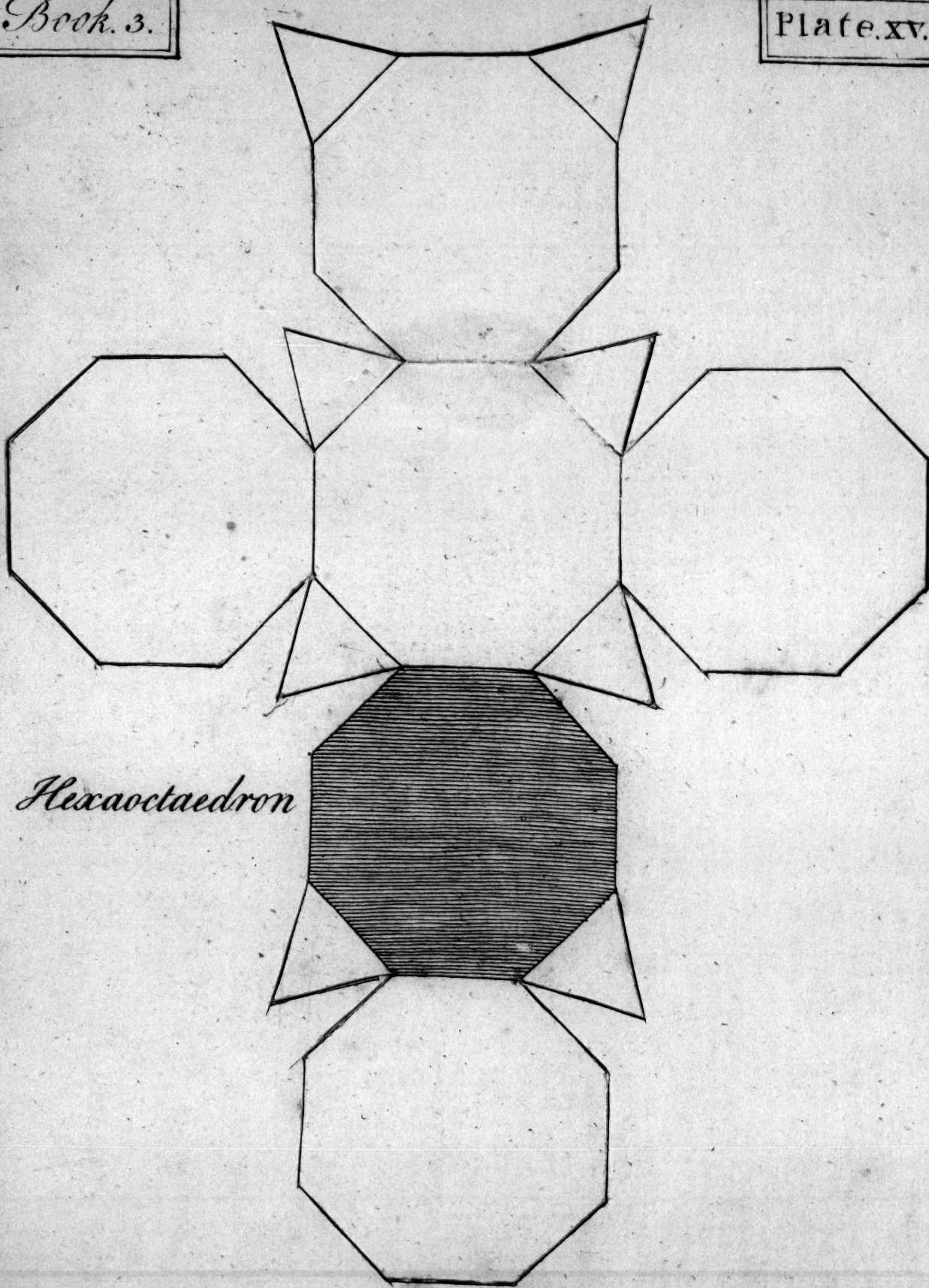


Book. 3.

Plate. XIV.





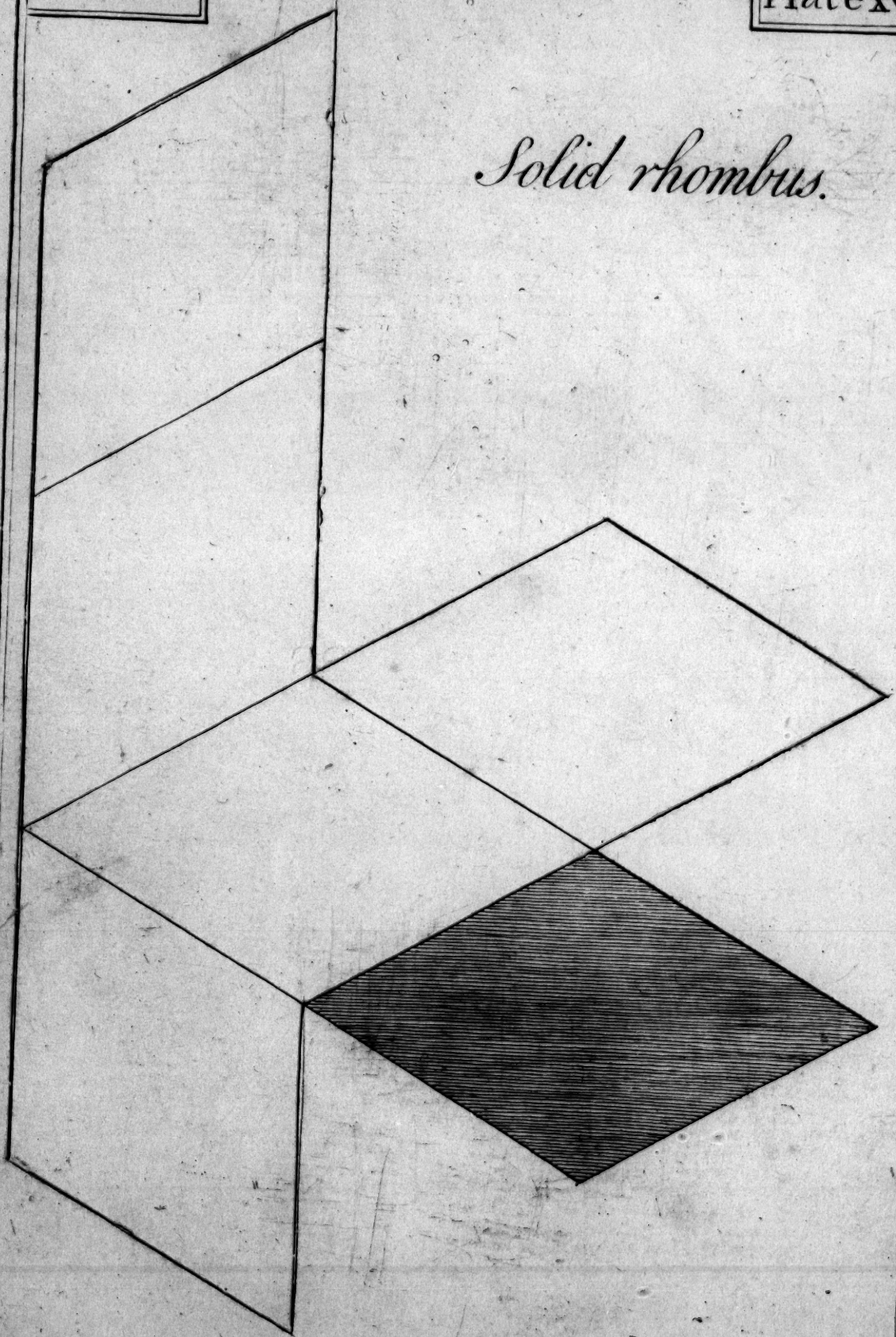


Hexaoctaedron

Book. 3.

Plate XVI

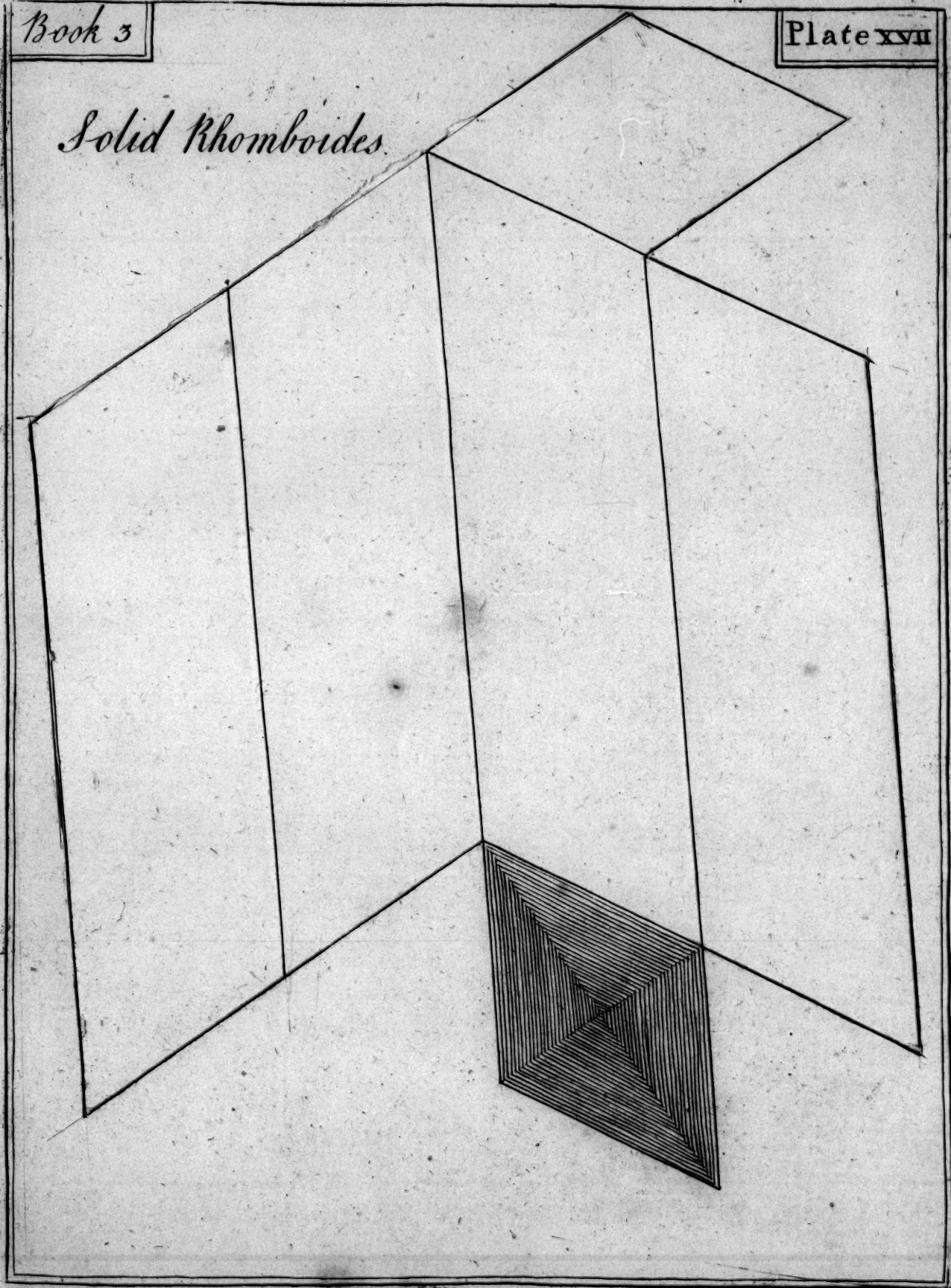
Solid rhombus.



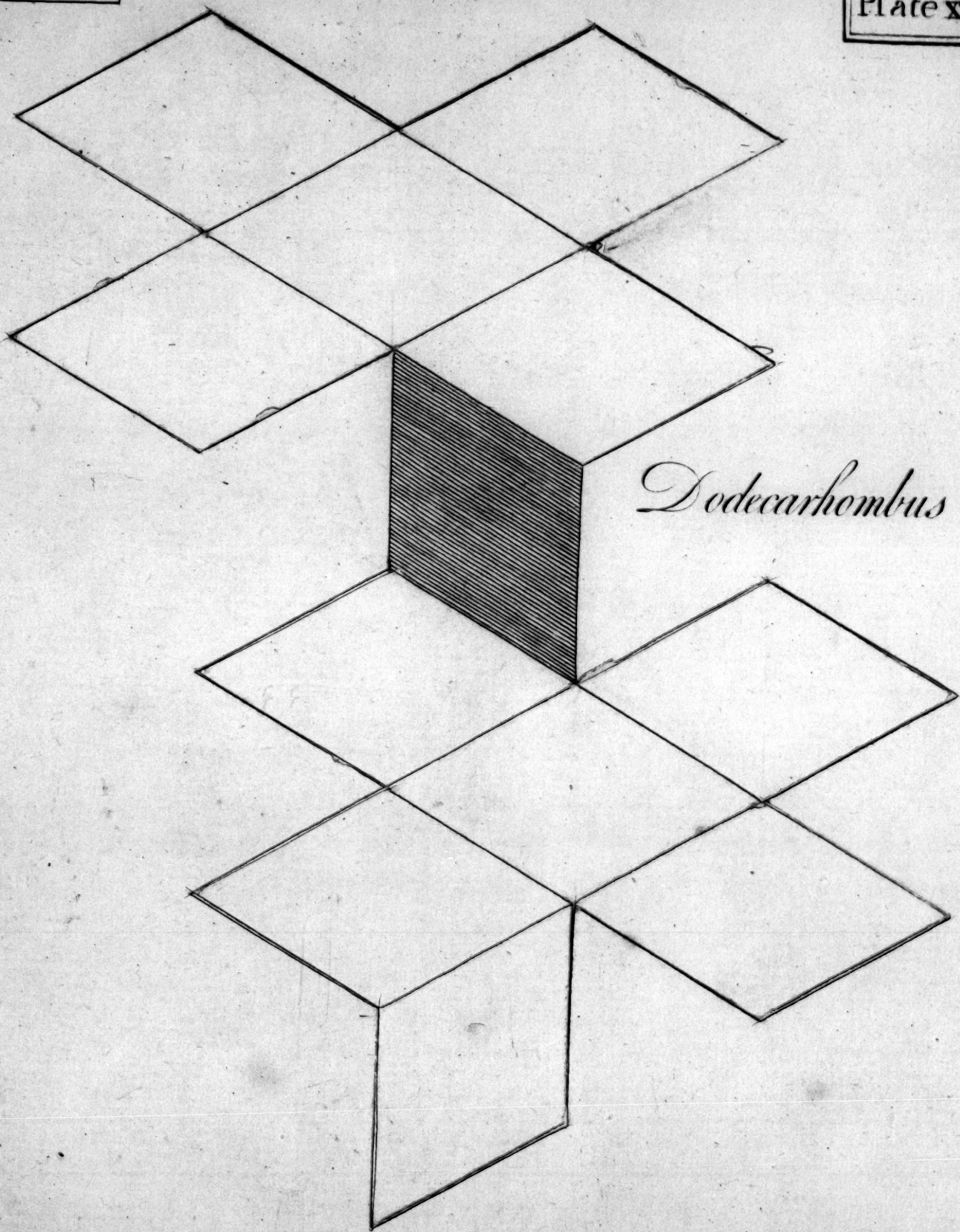
Book 3

Plate XVII

Solid Rhomboides.







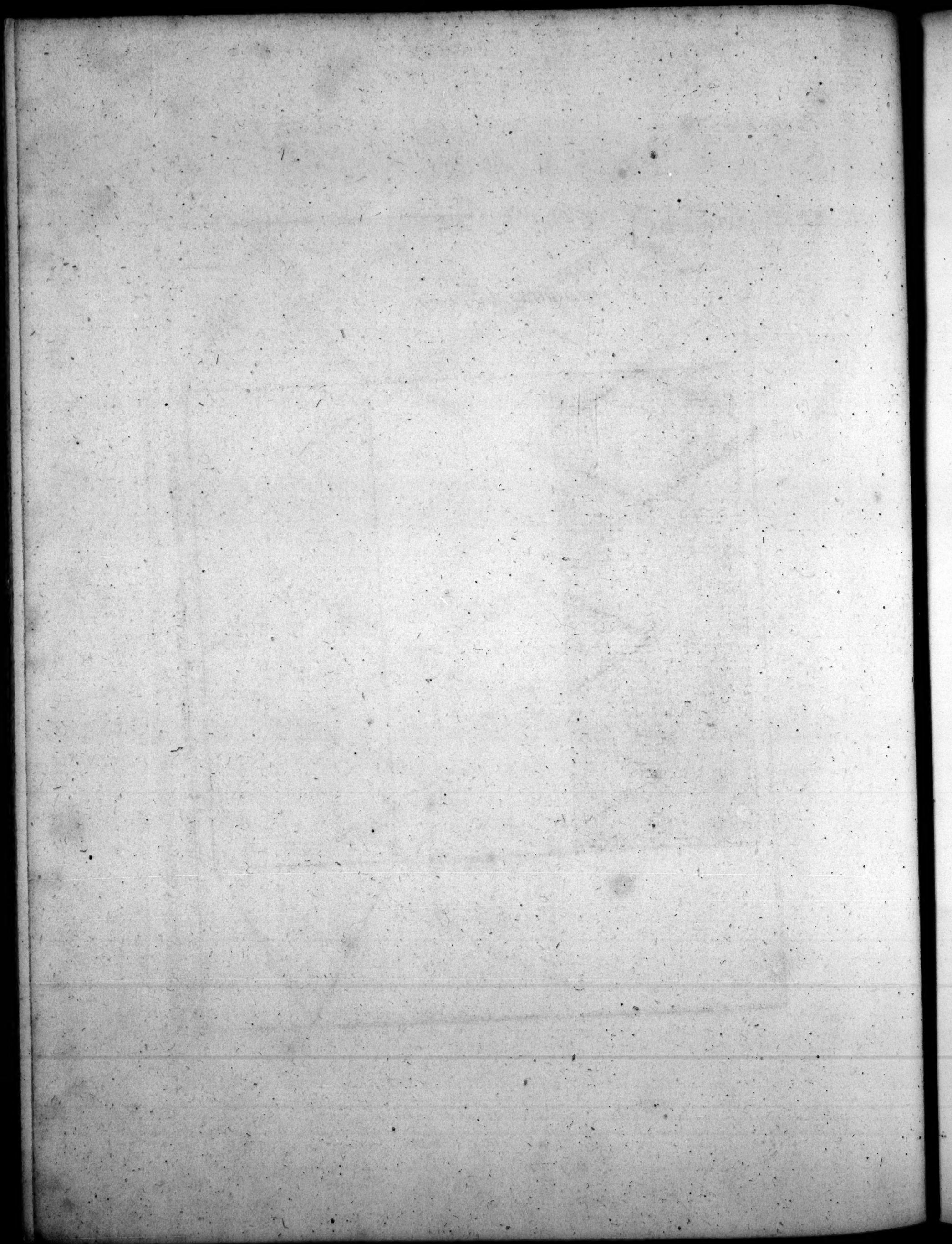
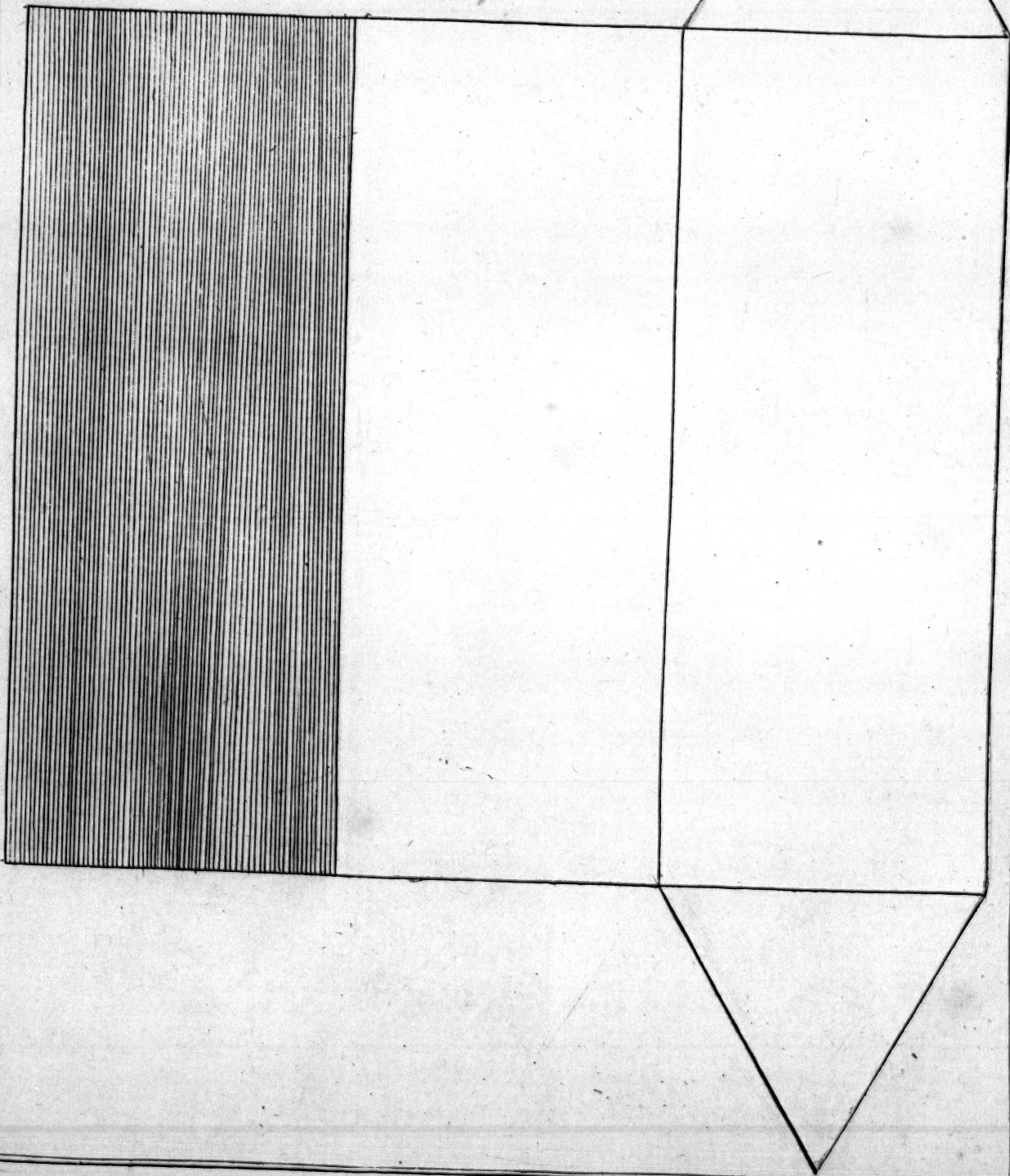


Plate XIX

Book 4

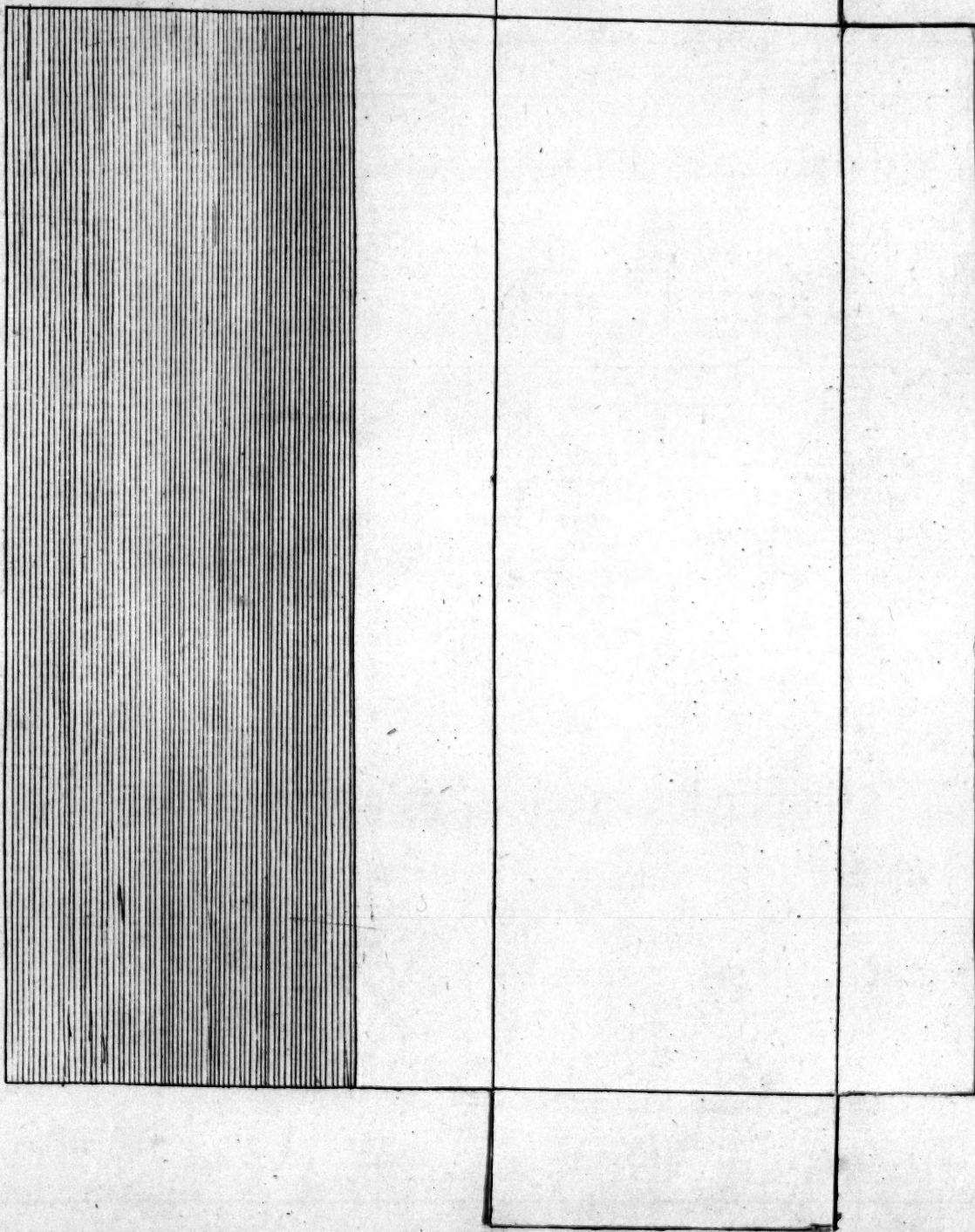
Triangular Prism

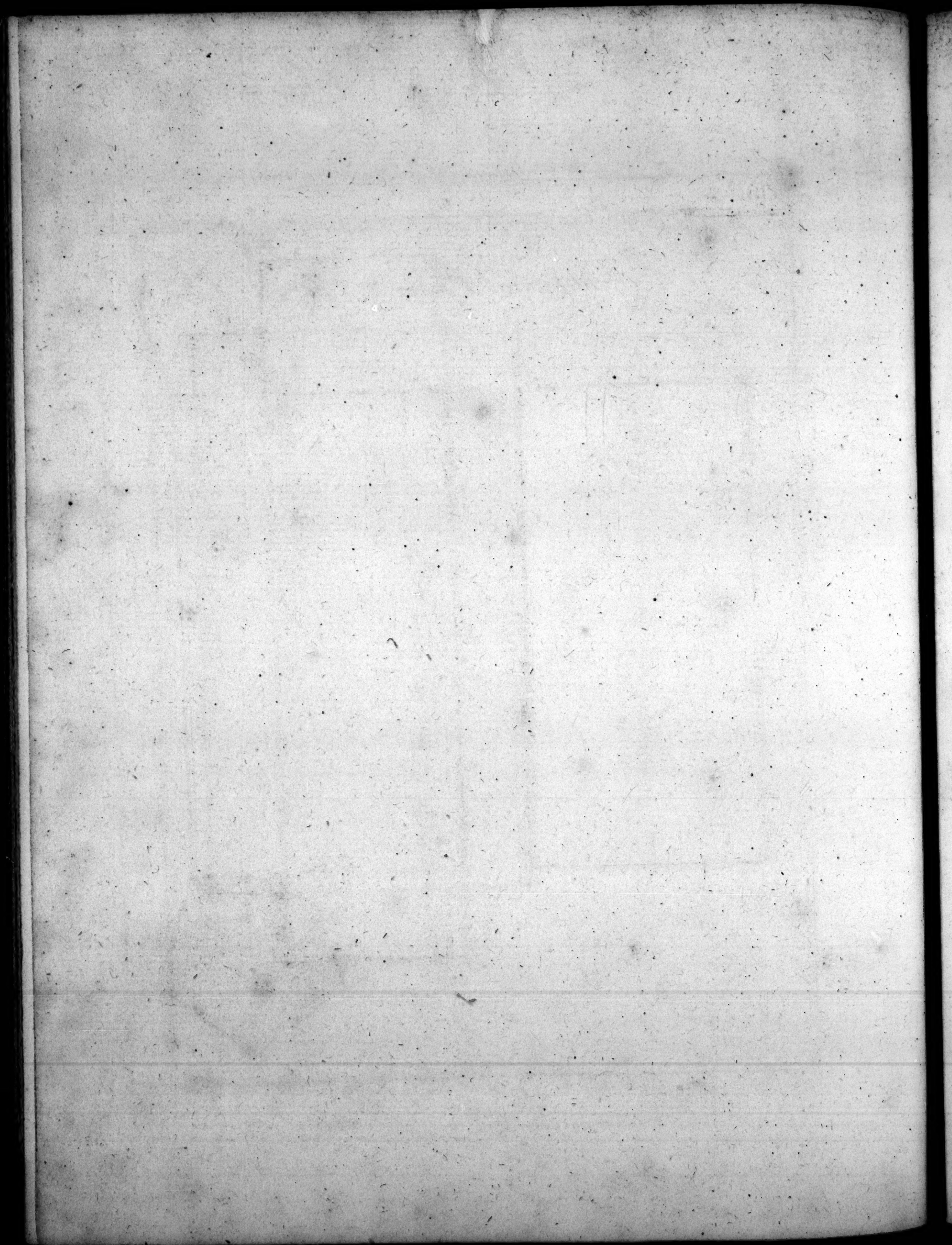


Book. 4.

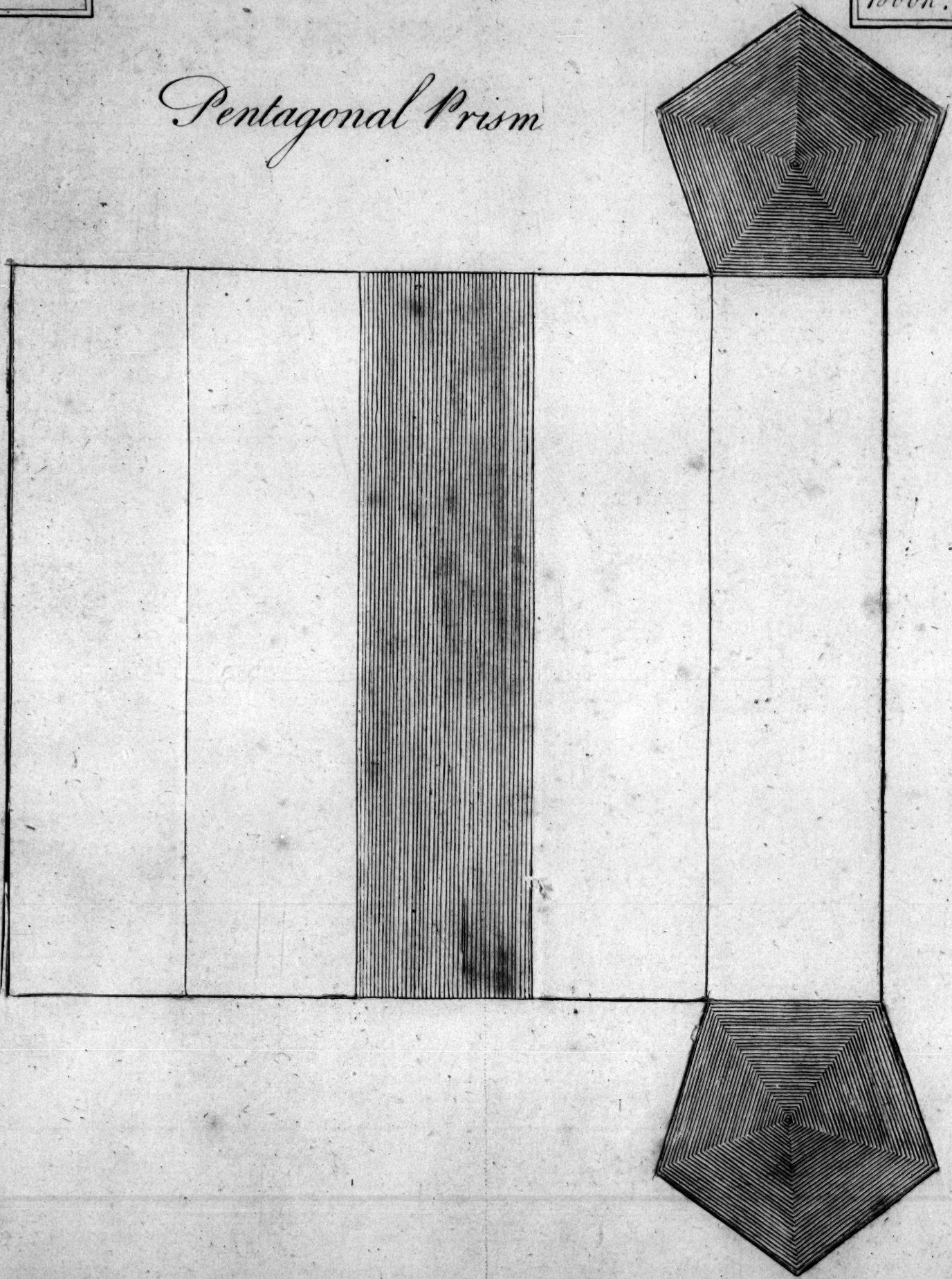
Plate XX

Parallelopipedon





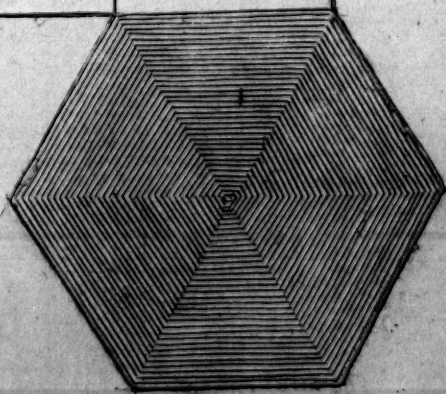
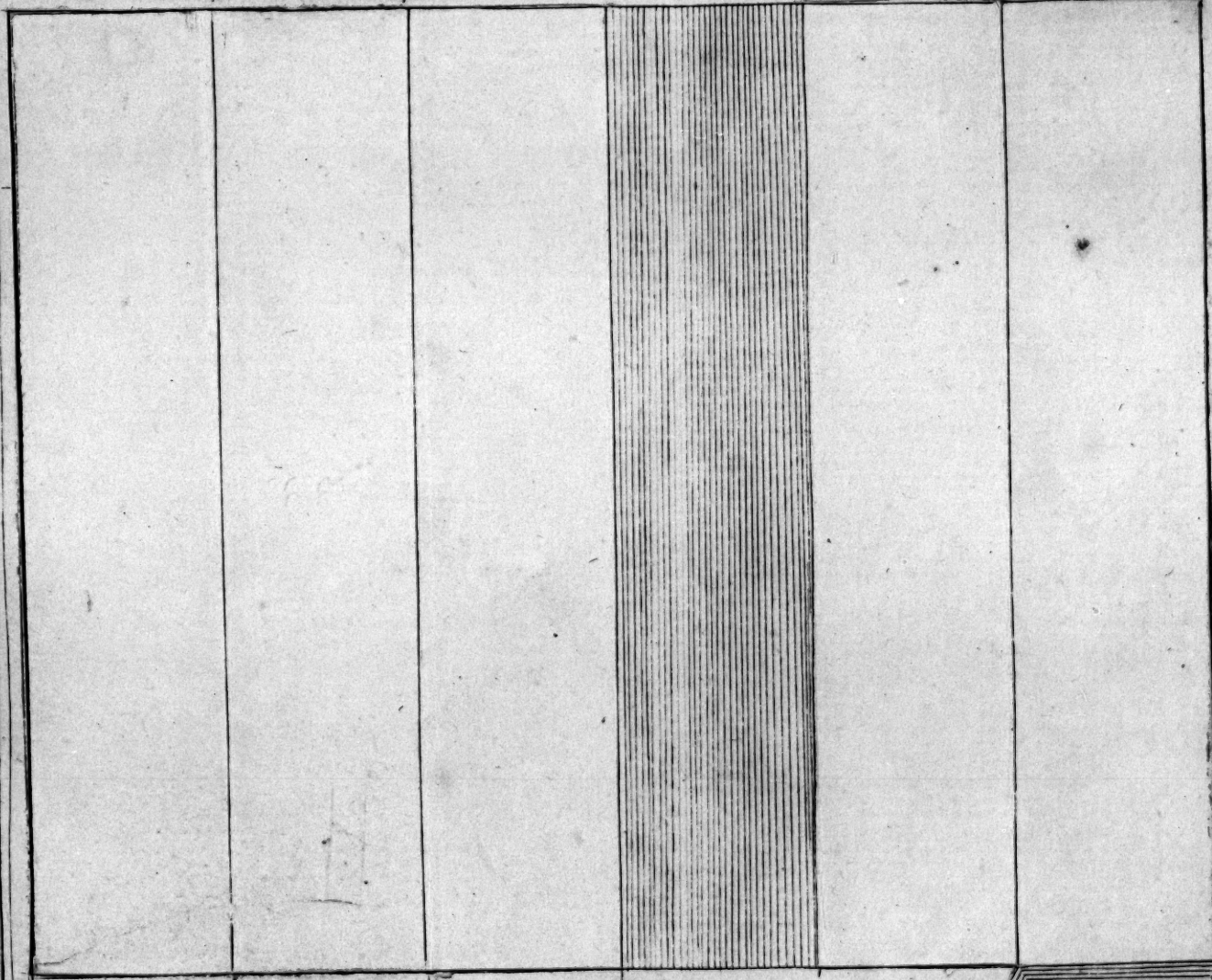
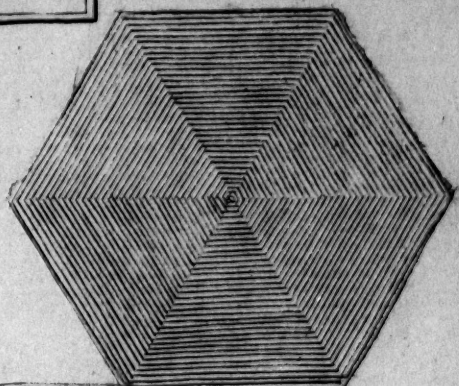
Pentagonal Prism

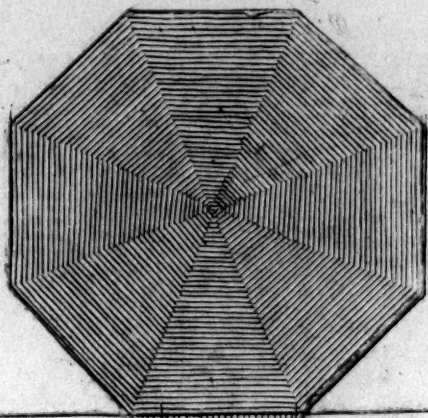


Platexxii

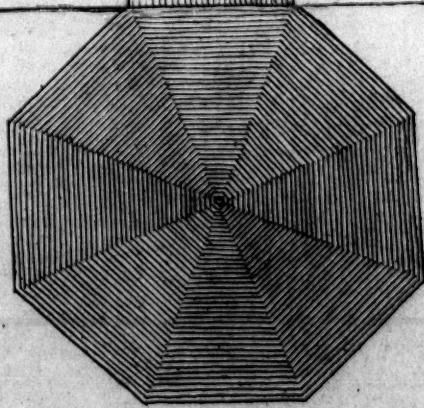
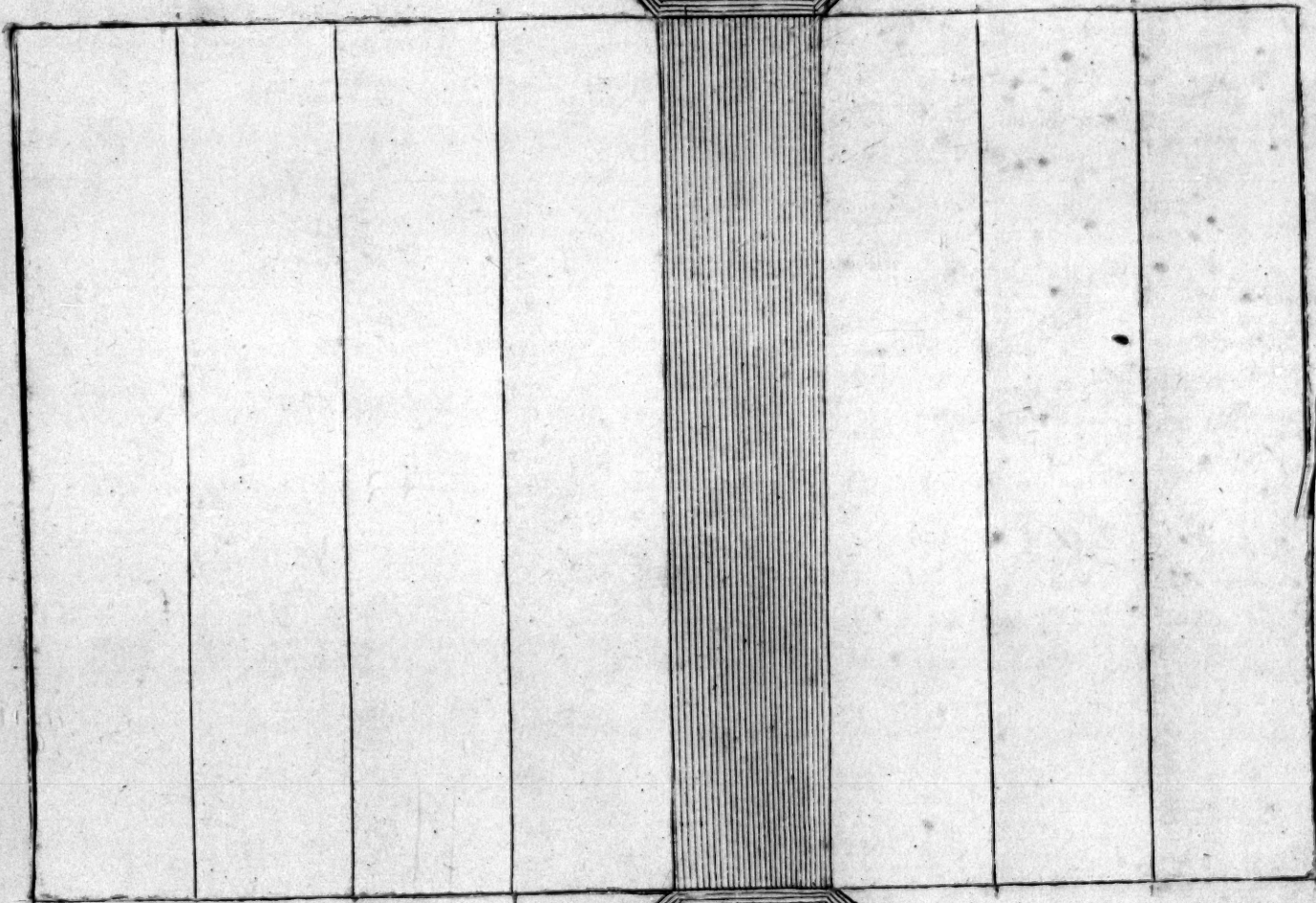
Book. 4.

Hexagonal Prism

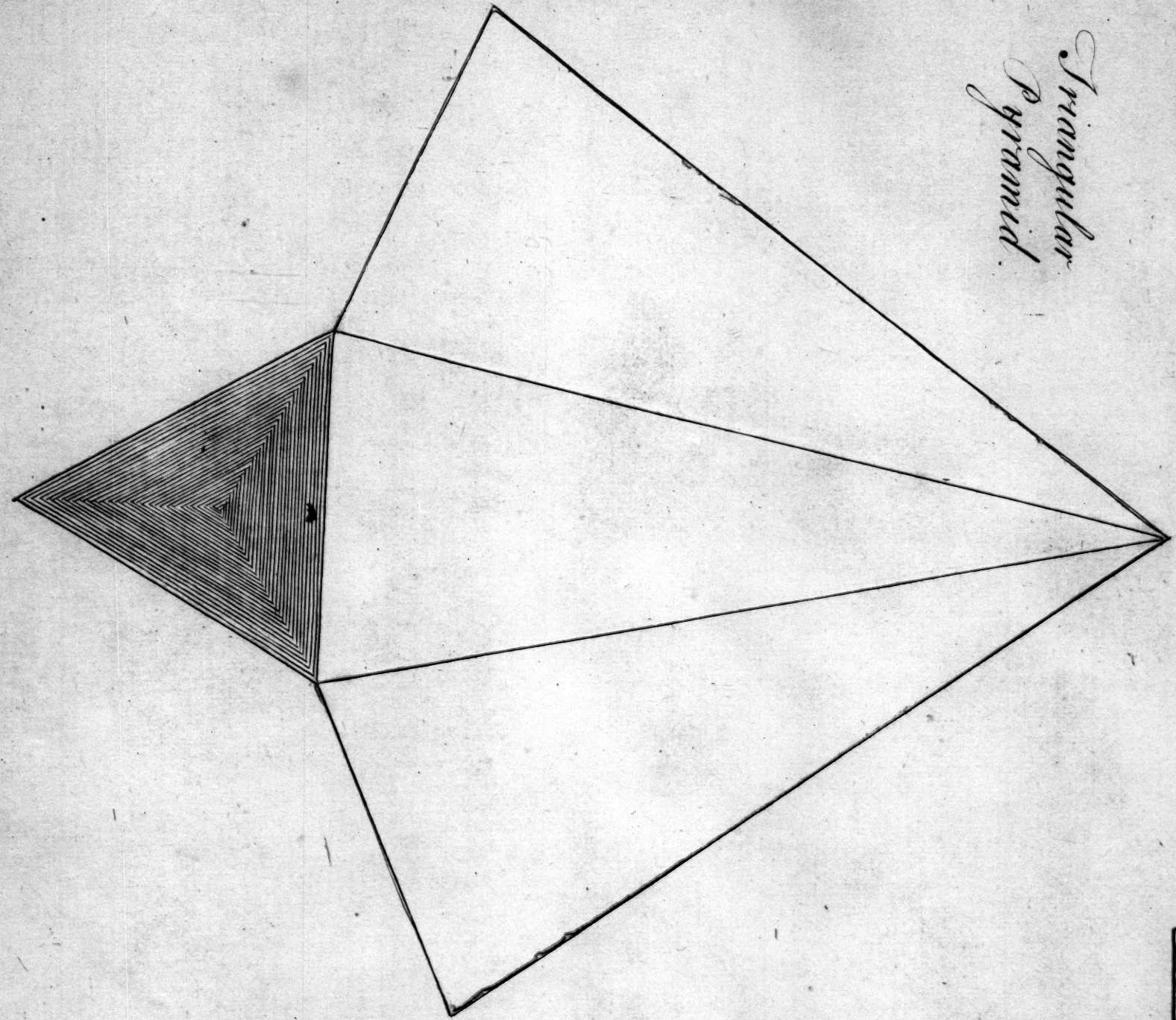


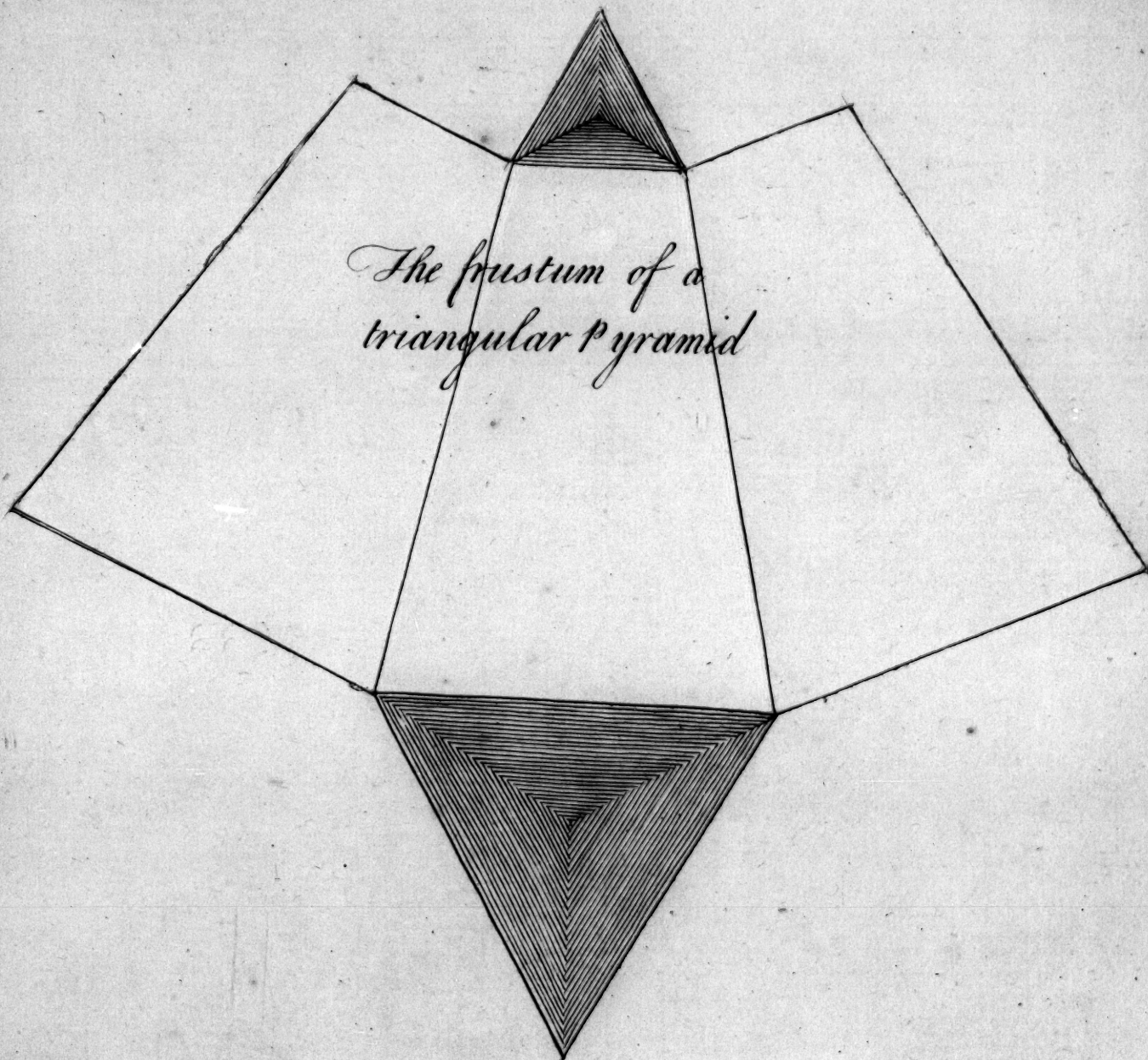


Octogonal Prism.



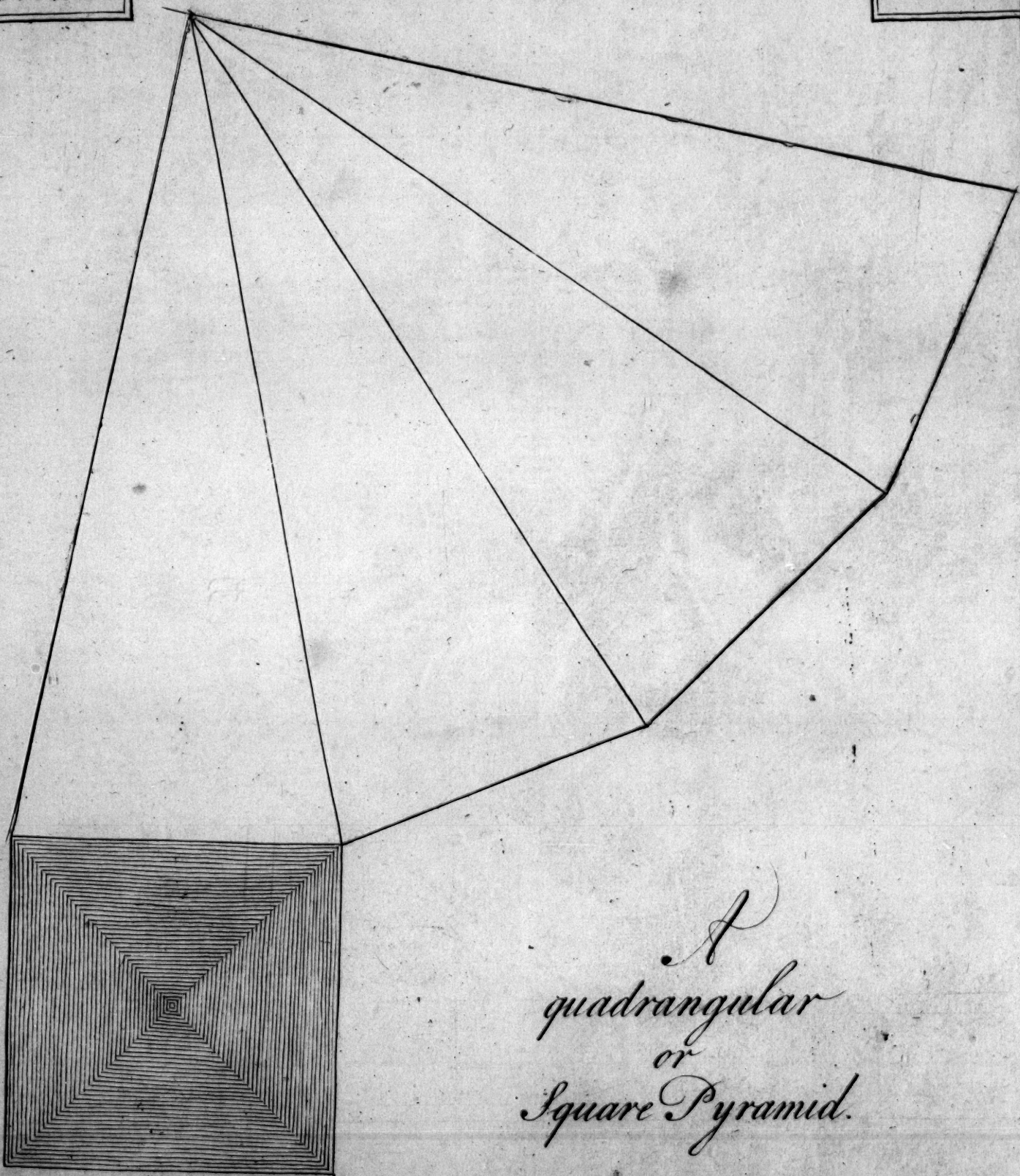
*Triangular
Pyramid.*





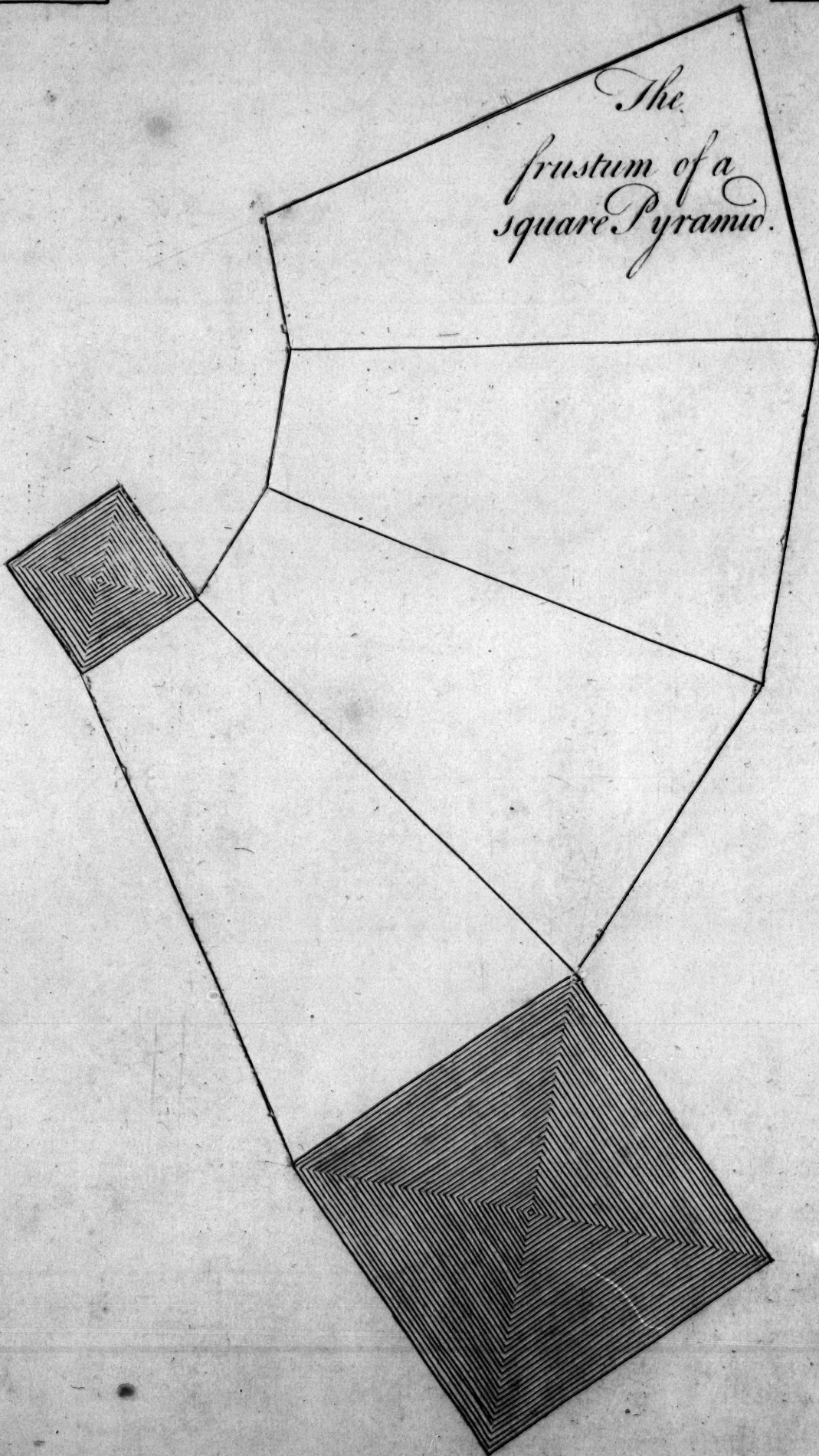
Book. 5.

Platexxvi

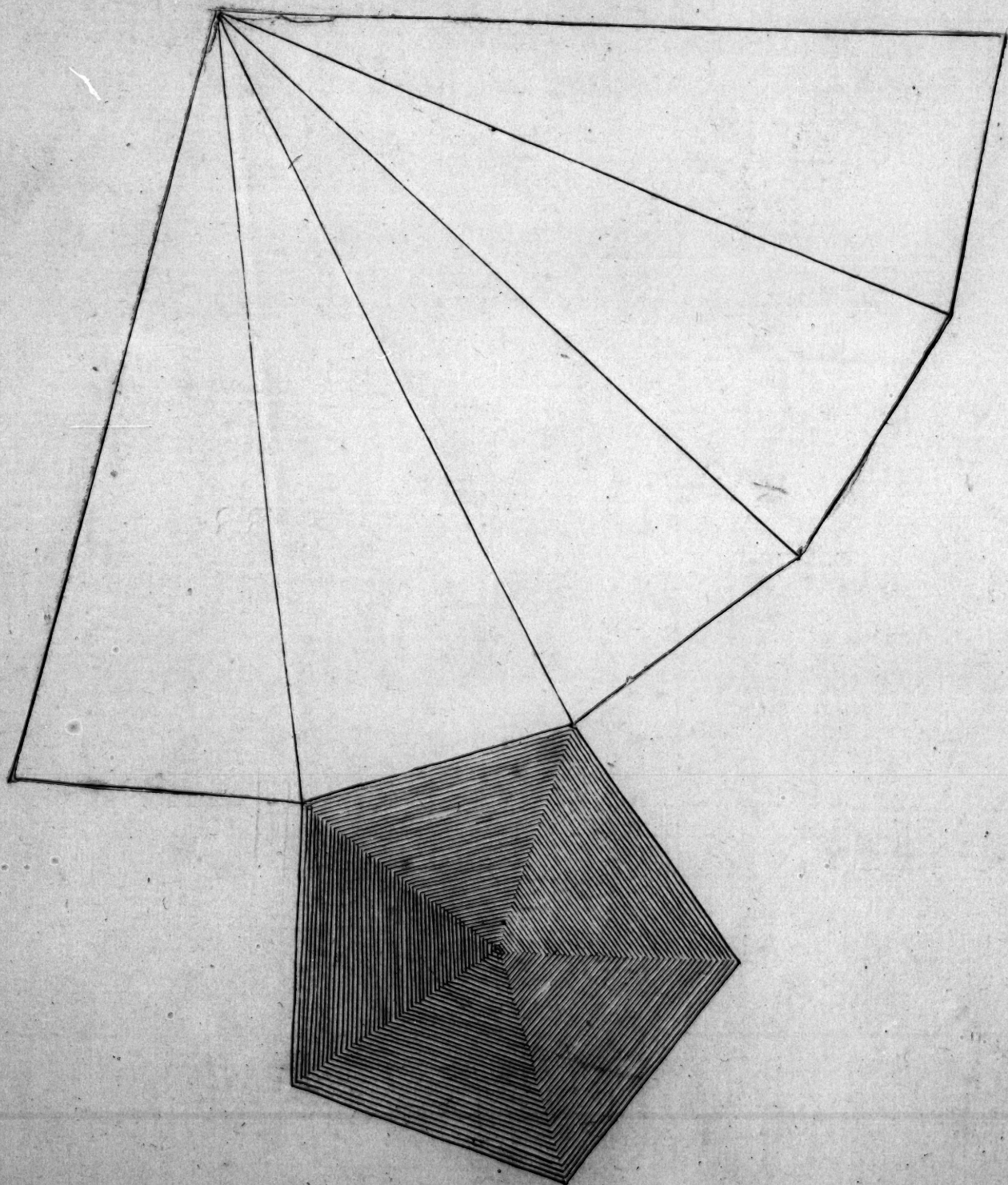


A
quadrangular
or
Square Pyramid.

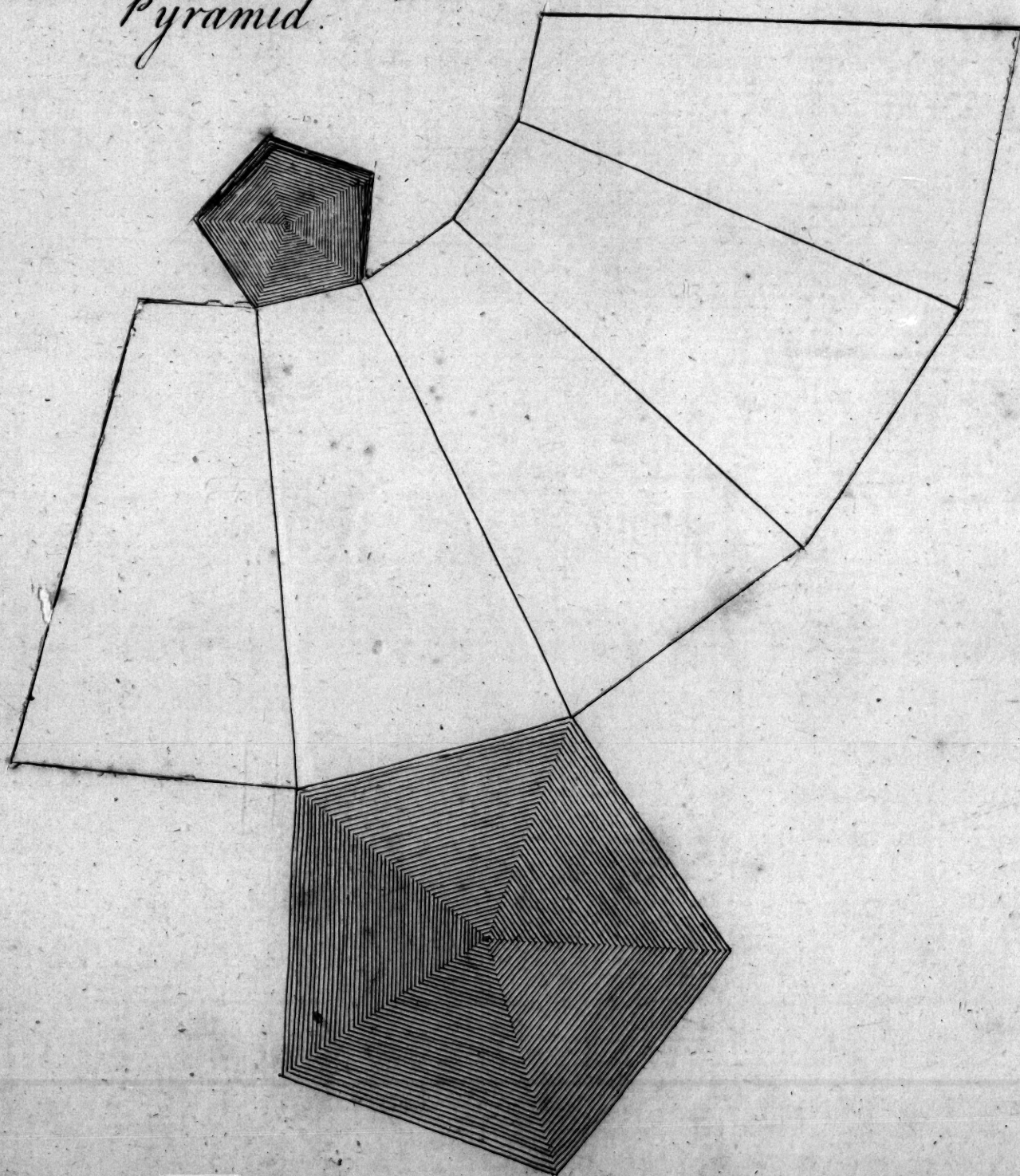
*The
frustum of a
square Pyramid.*



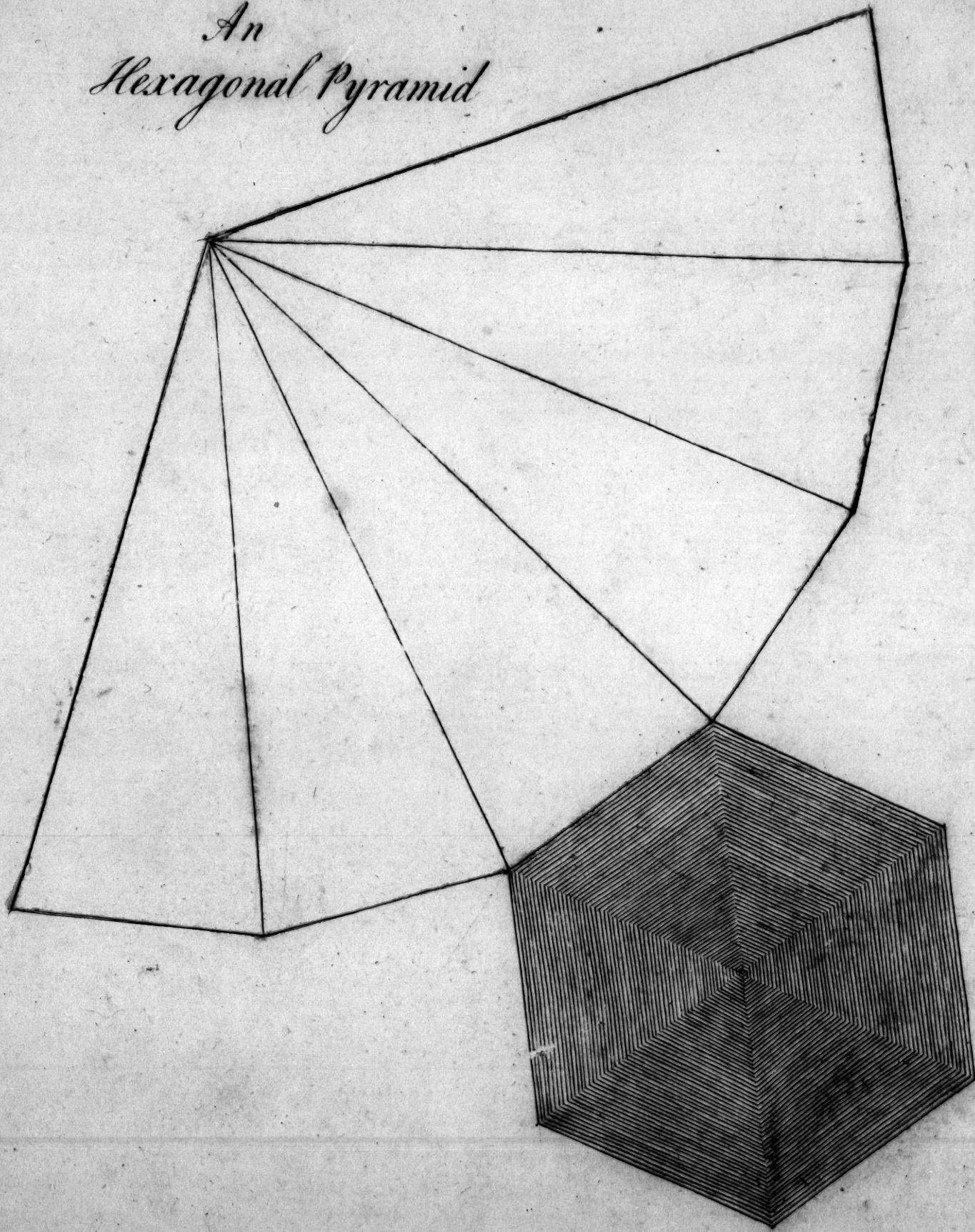
*A
Pentagonal Pyramid.*

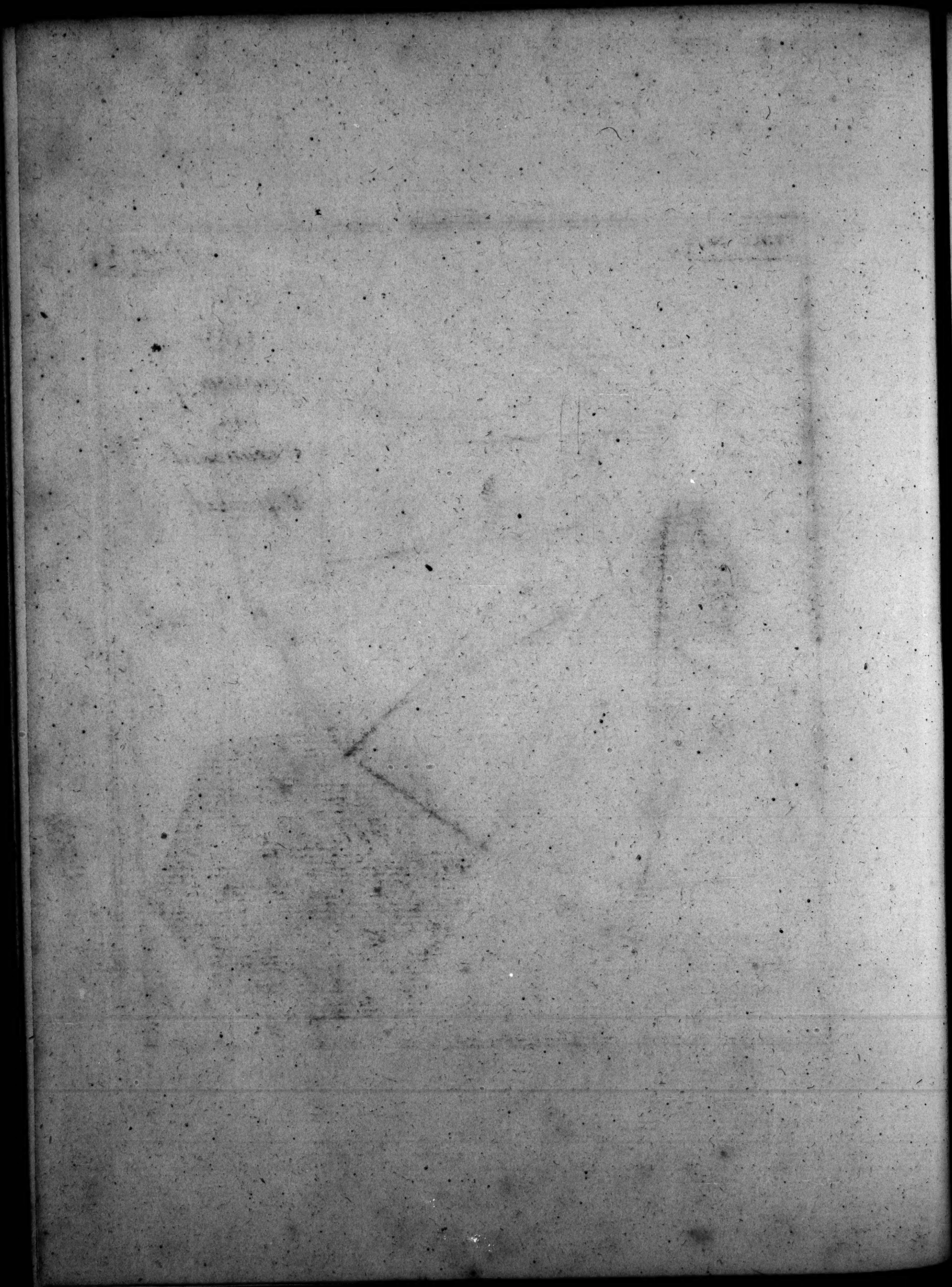


*The
frustum of a Pentagonal
Pyramid.*

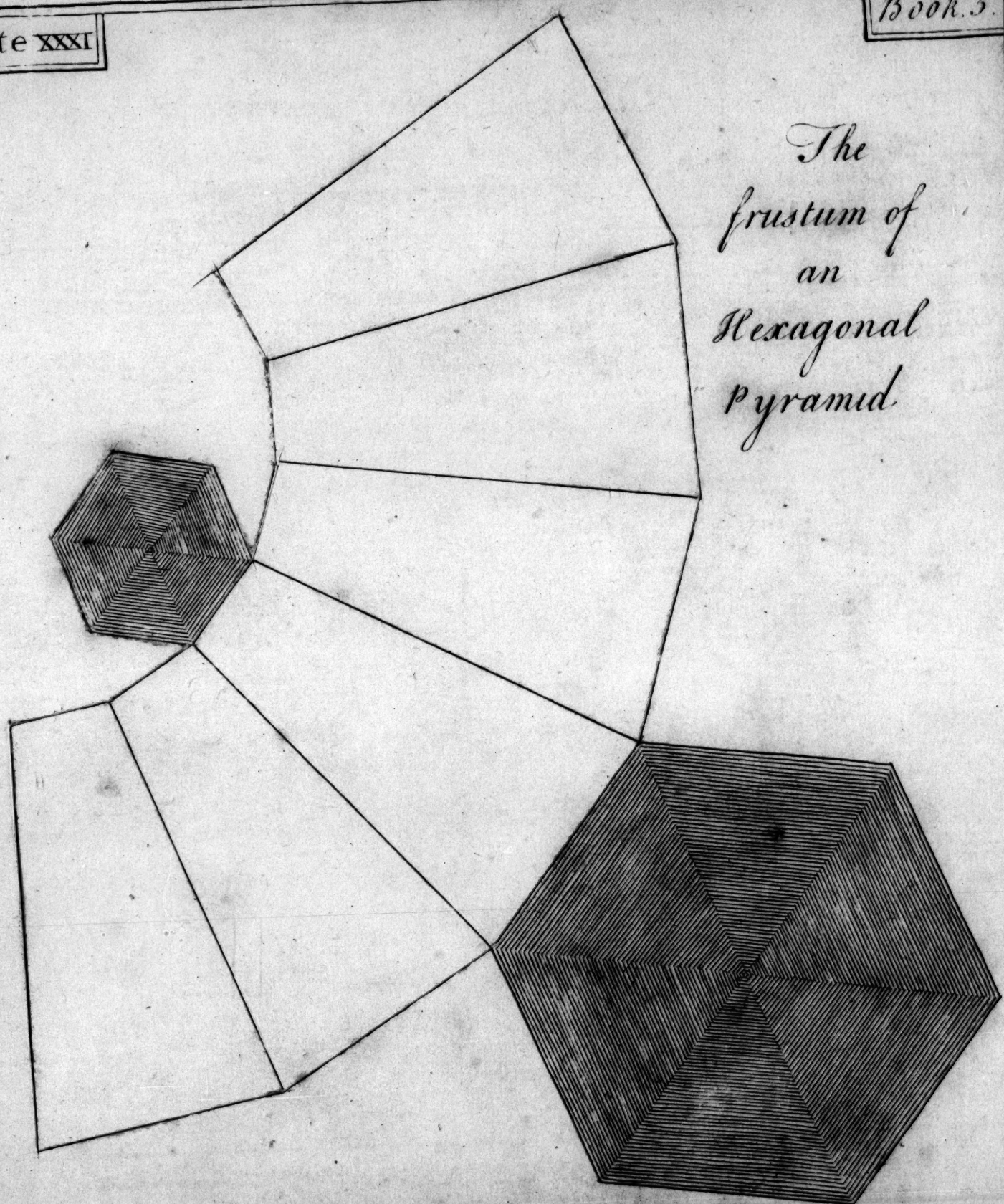


*An
Hexagonal Pyramid*

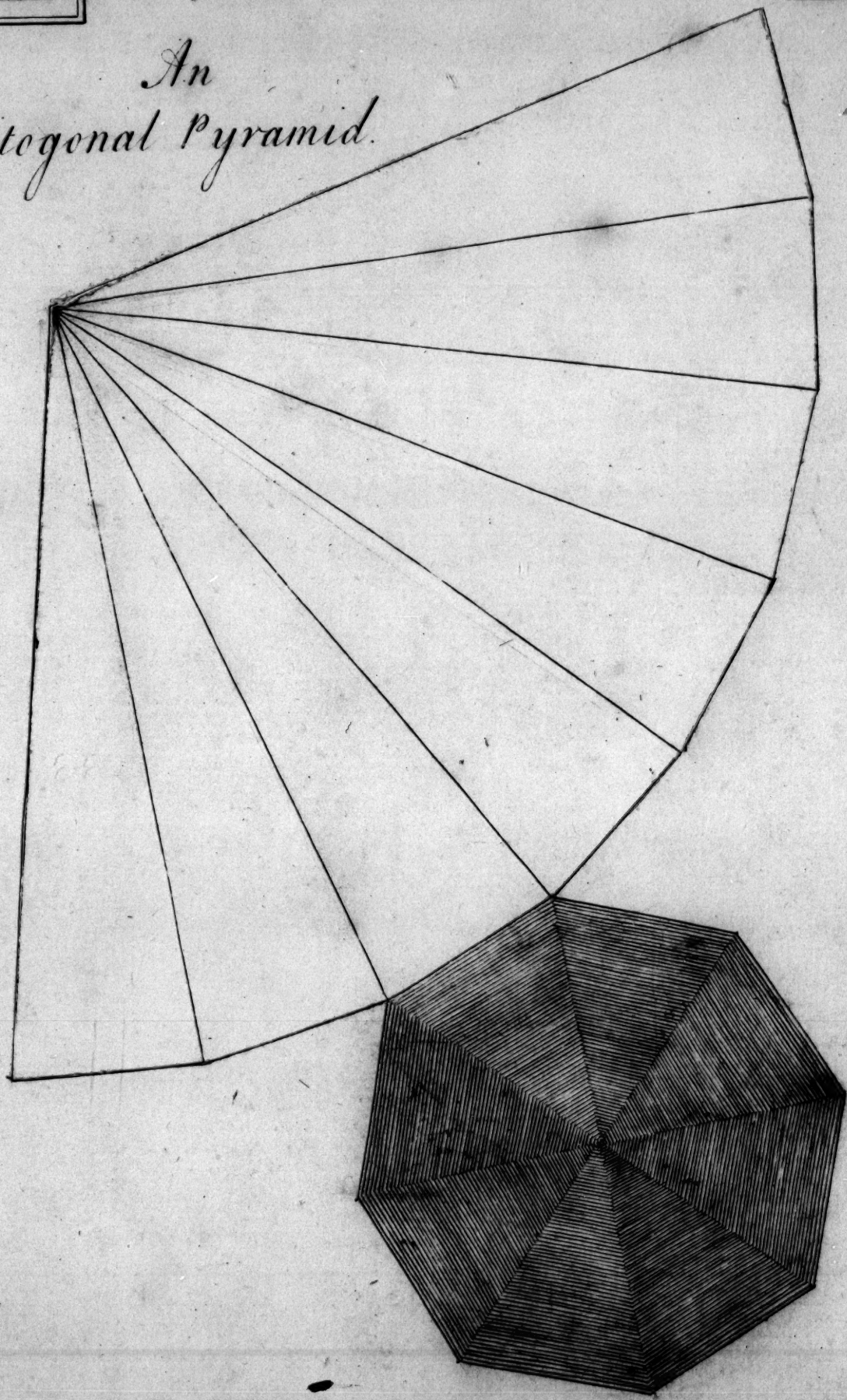




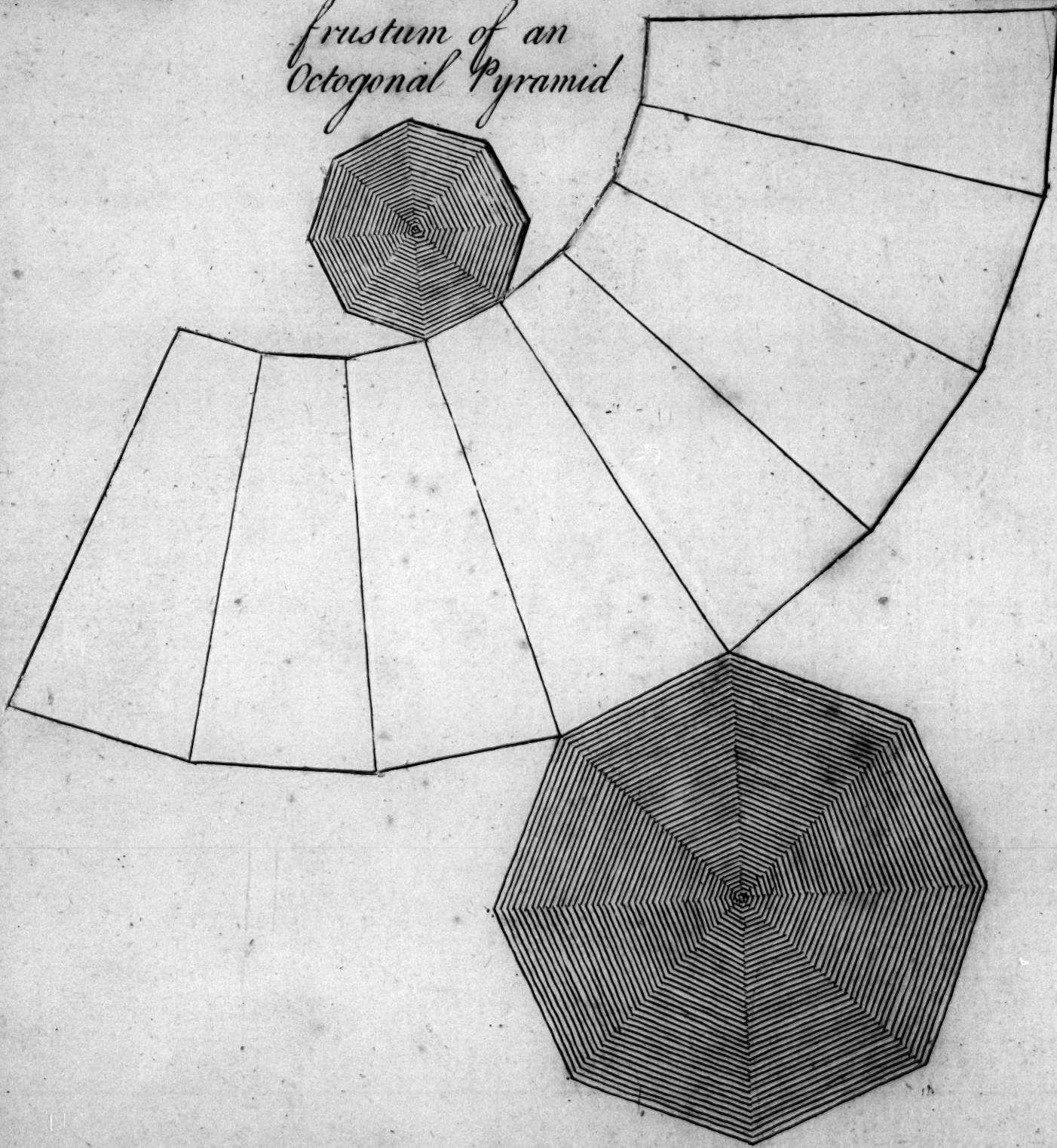
*The
frustum of
an
Hexagonal
pyramid*

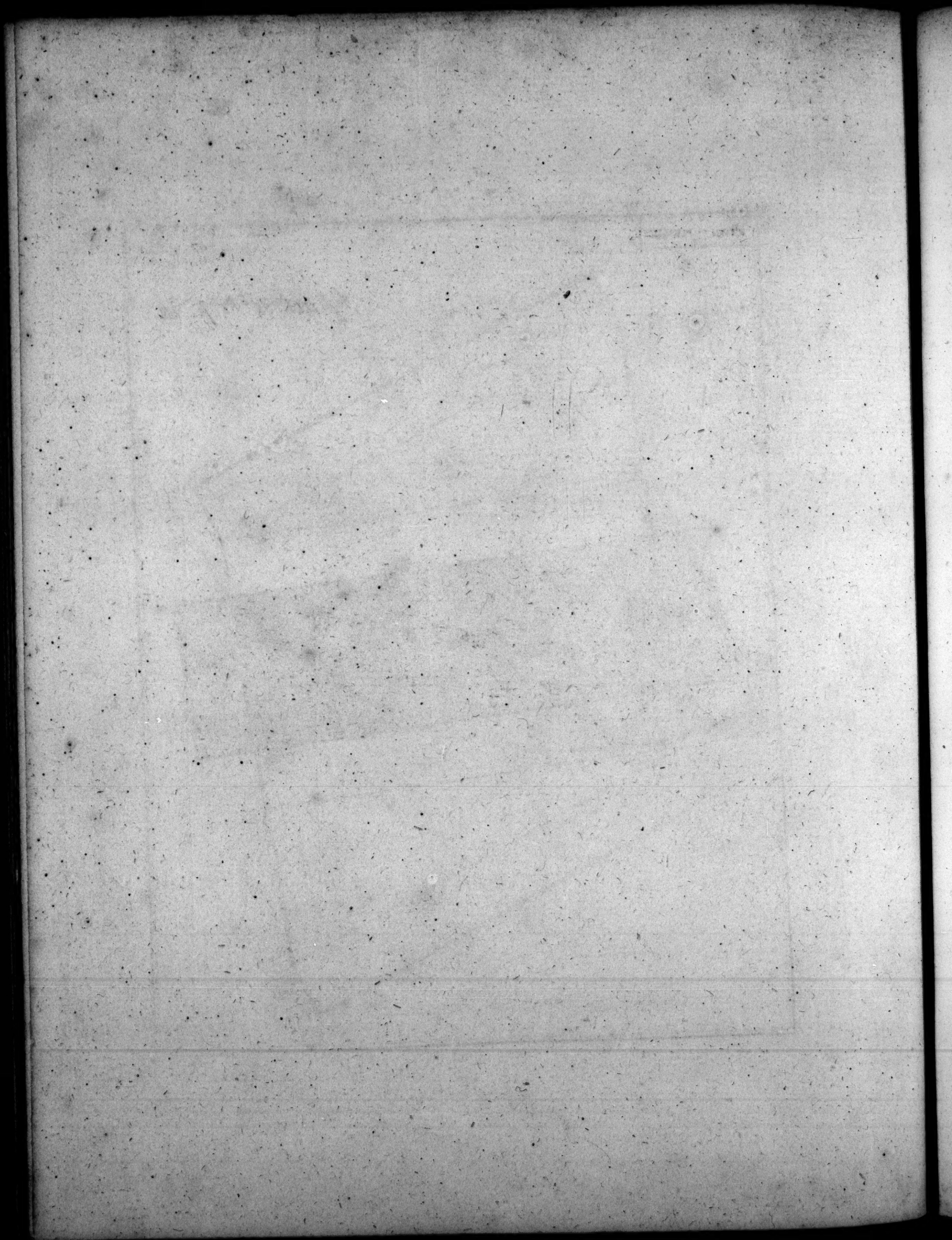


*An
Octogonal Pyramid.*



*The
frustum of an
Octagonal Pyramid*





Eucl. XI. Prop. 28.

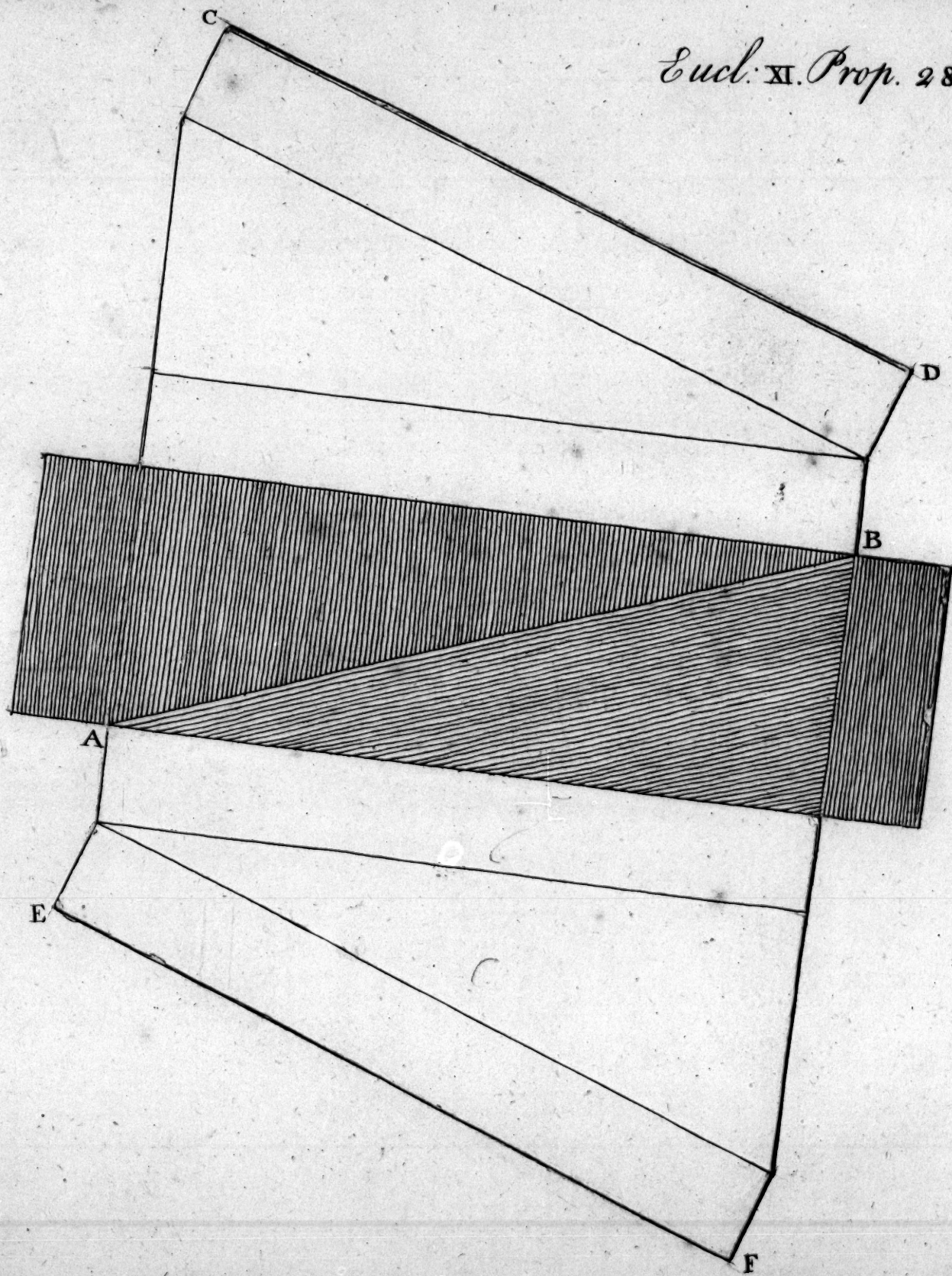
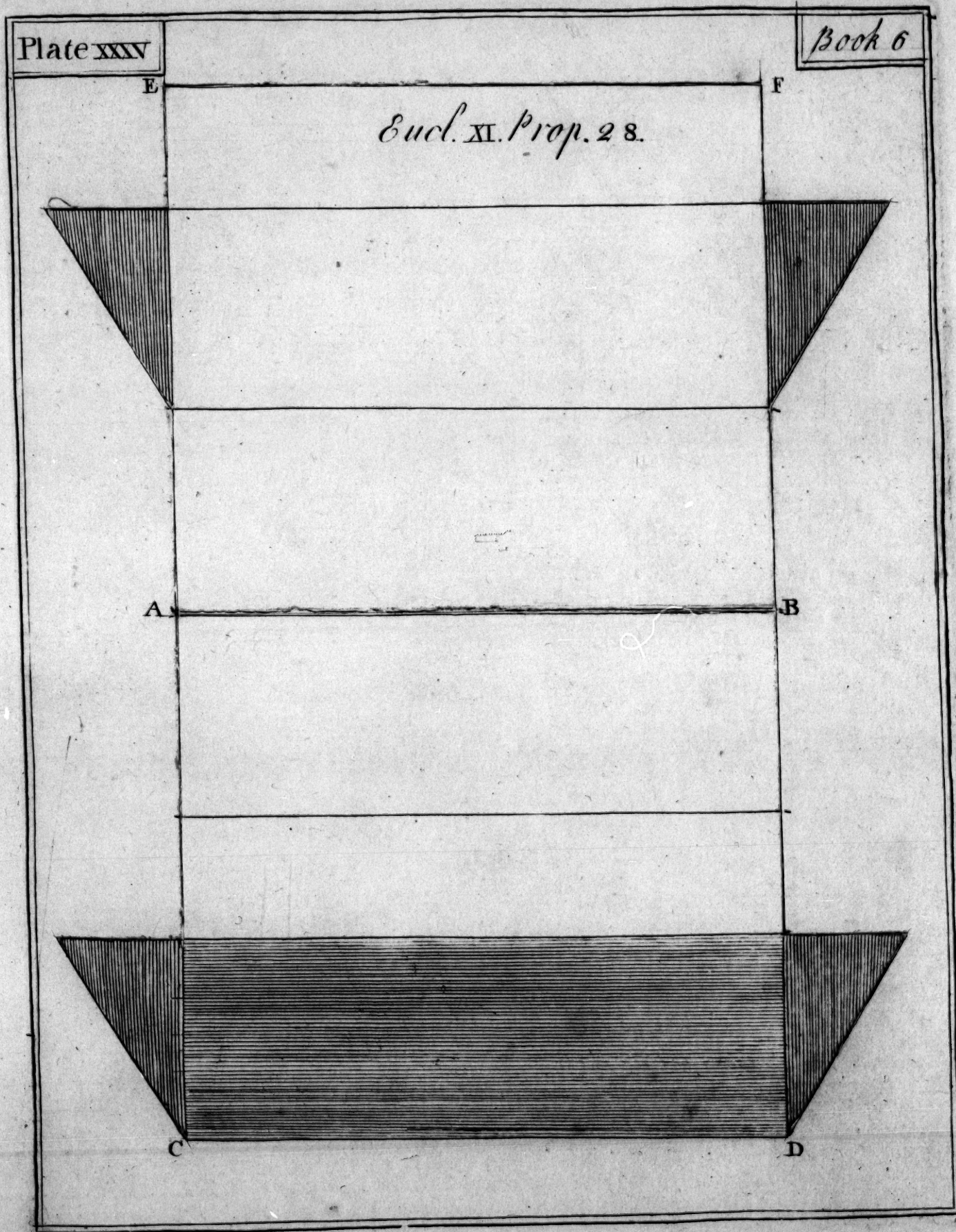
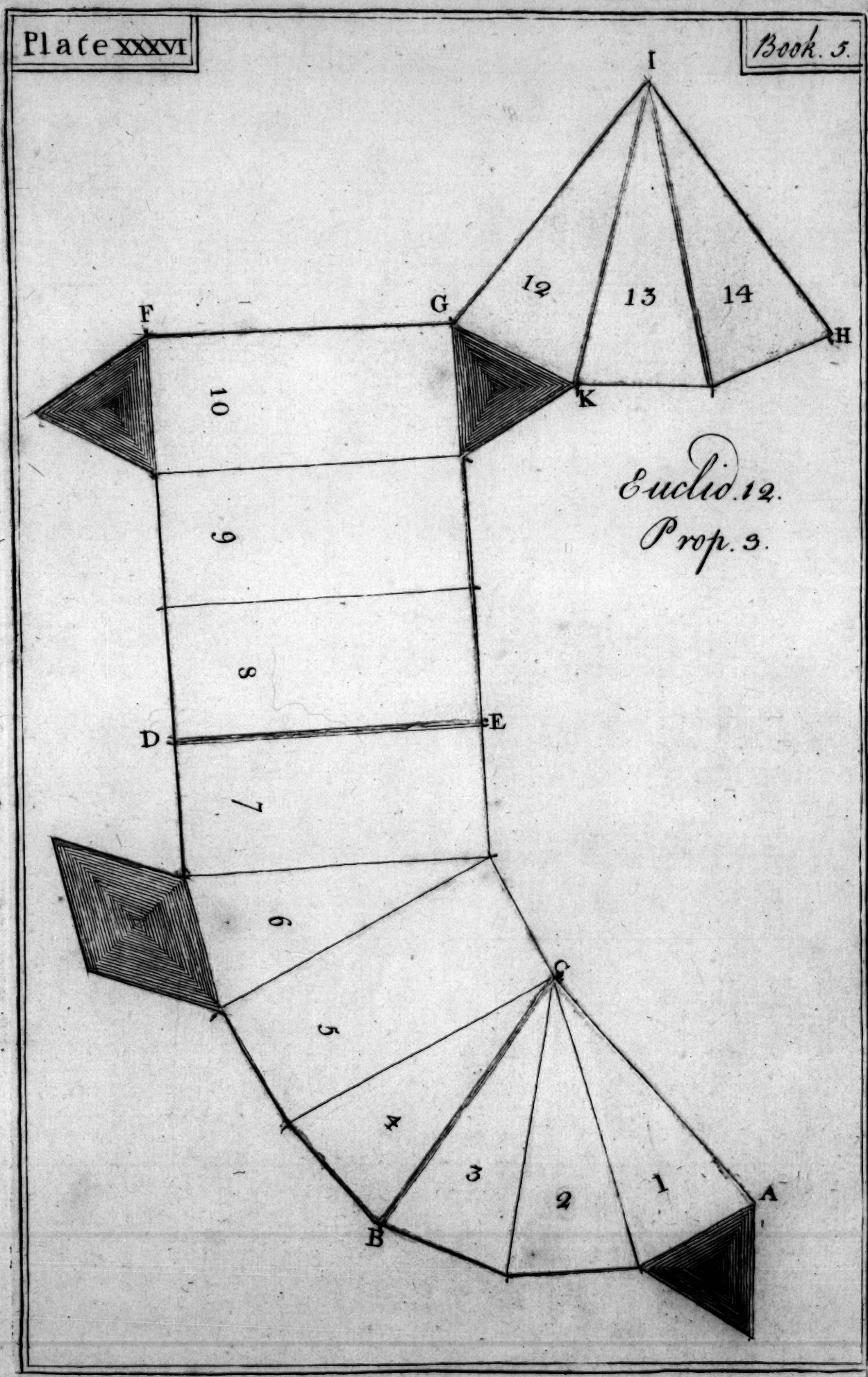


Plate XXXV

Book 6

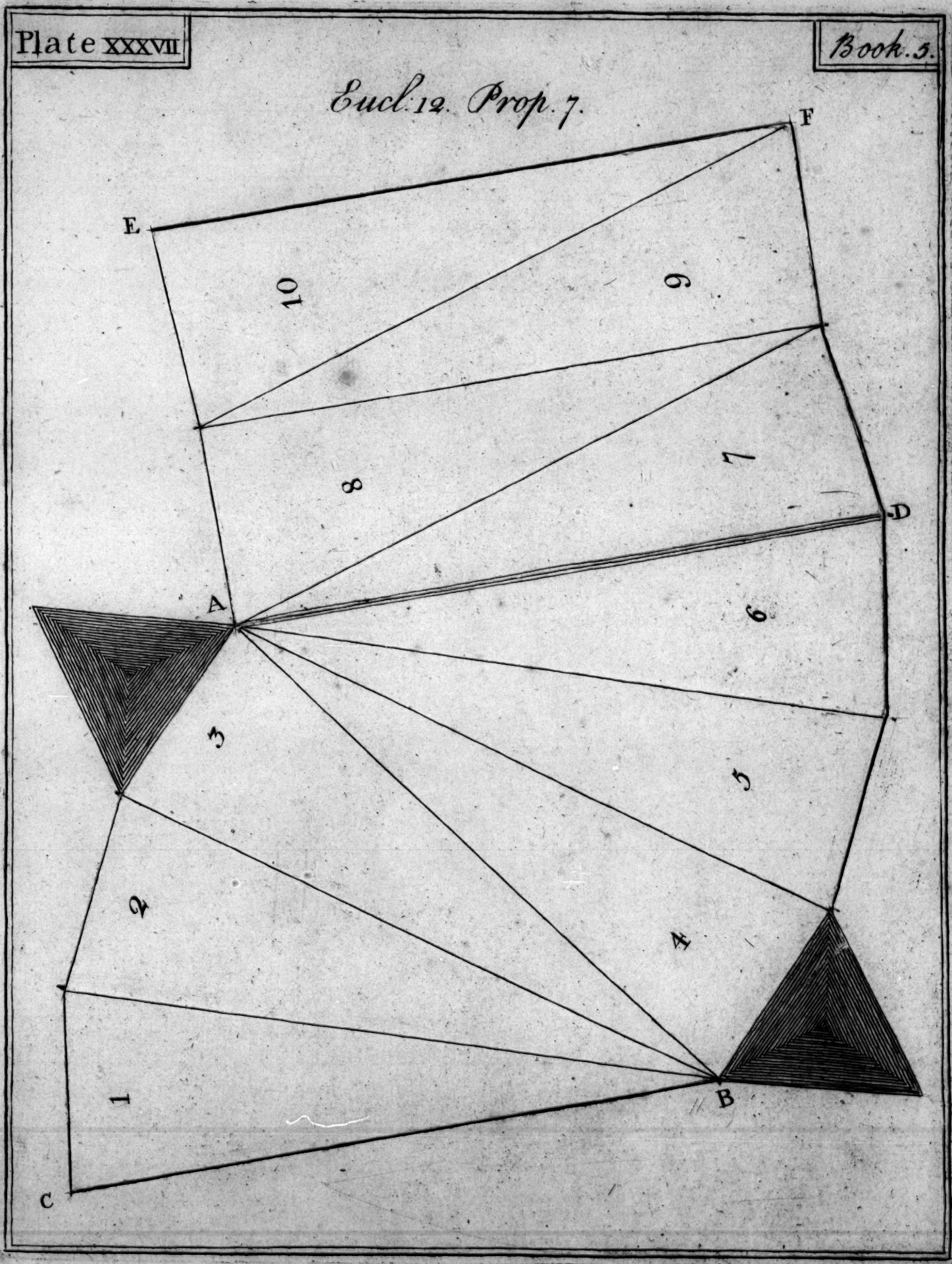
Eucl. XI. Prop. 28.

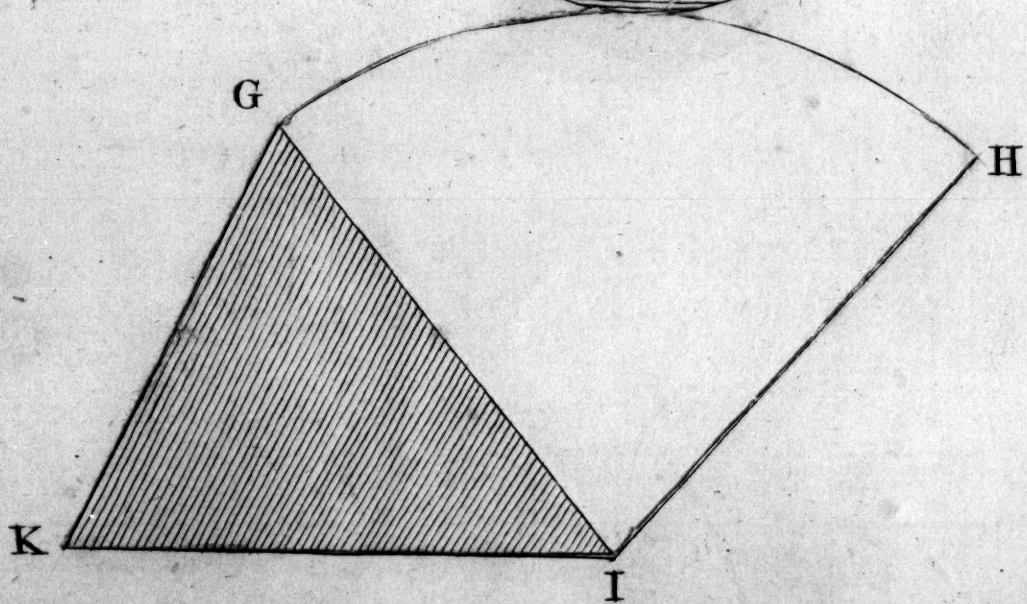
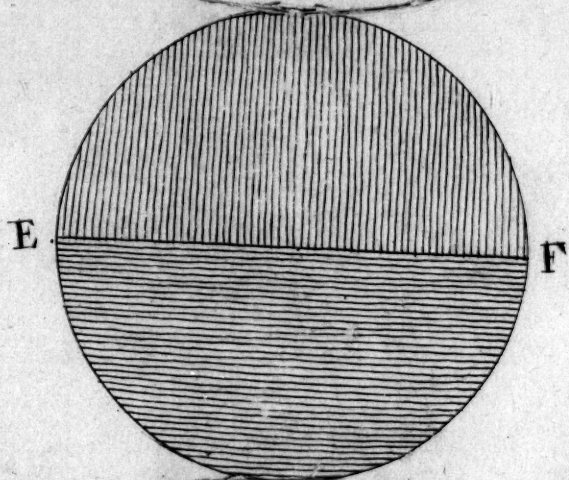
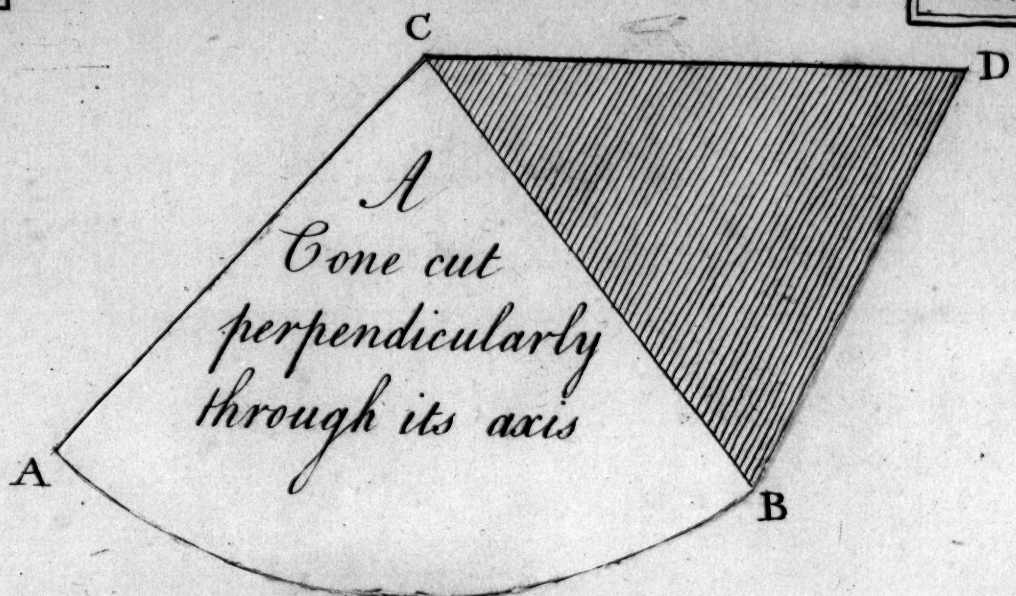


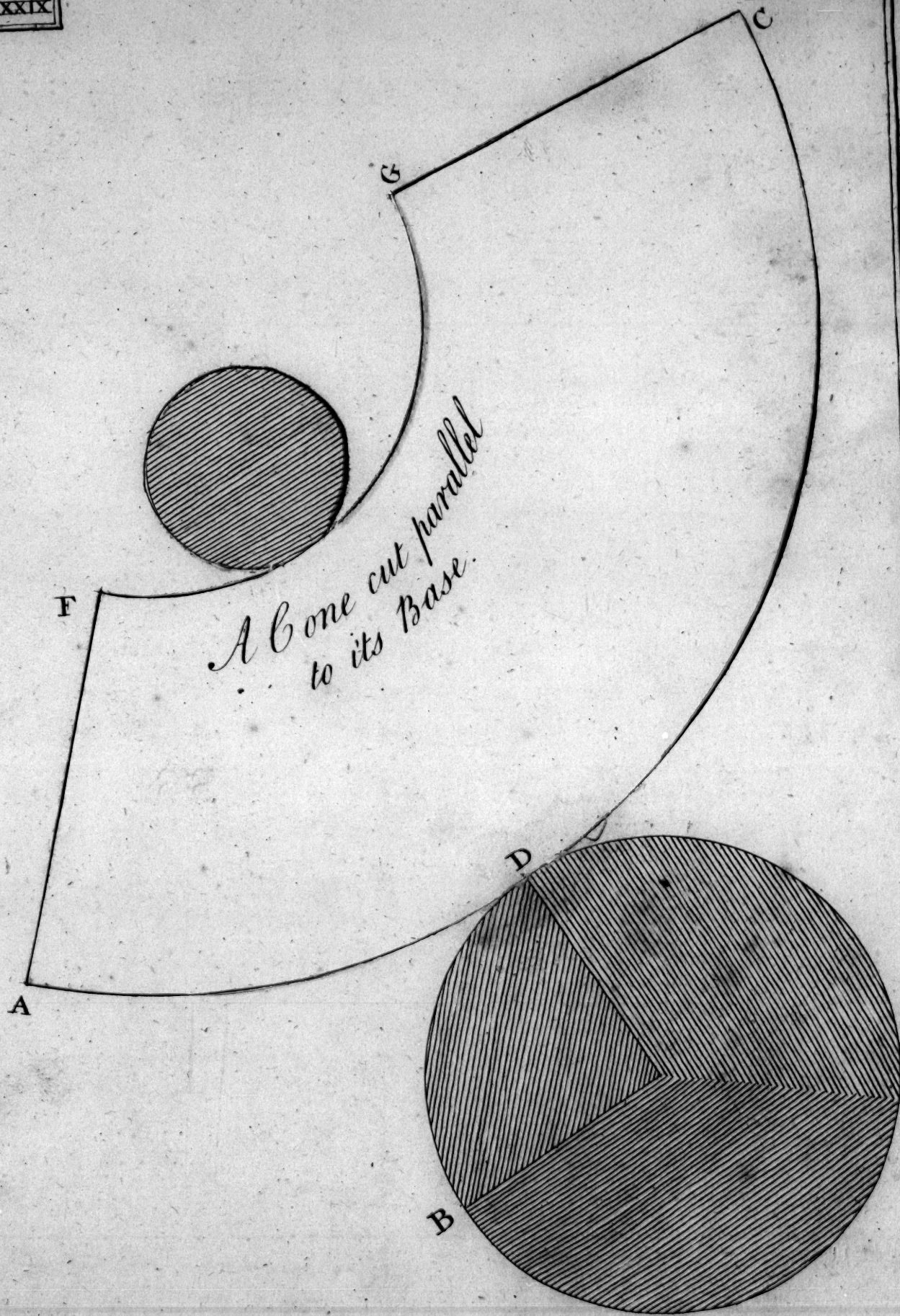


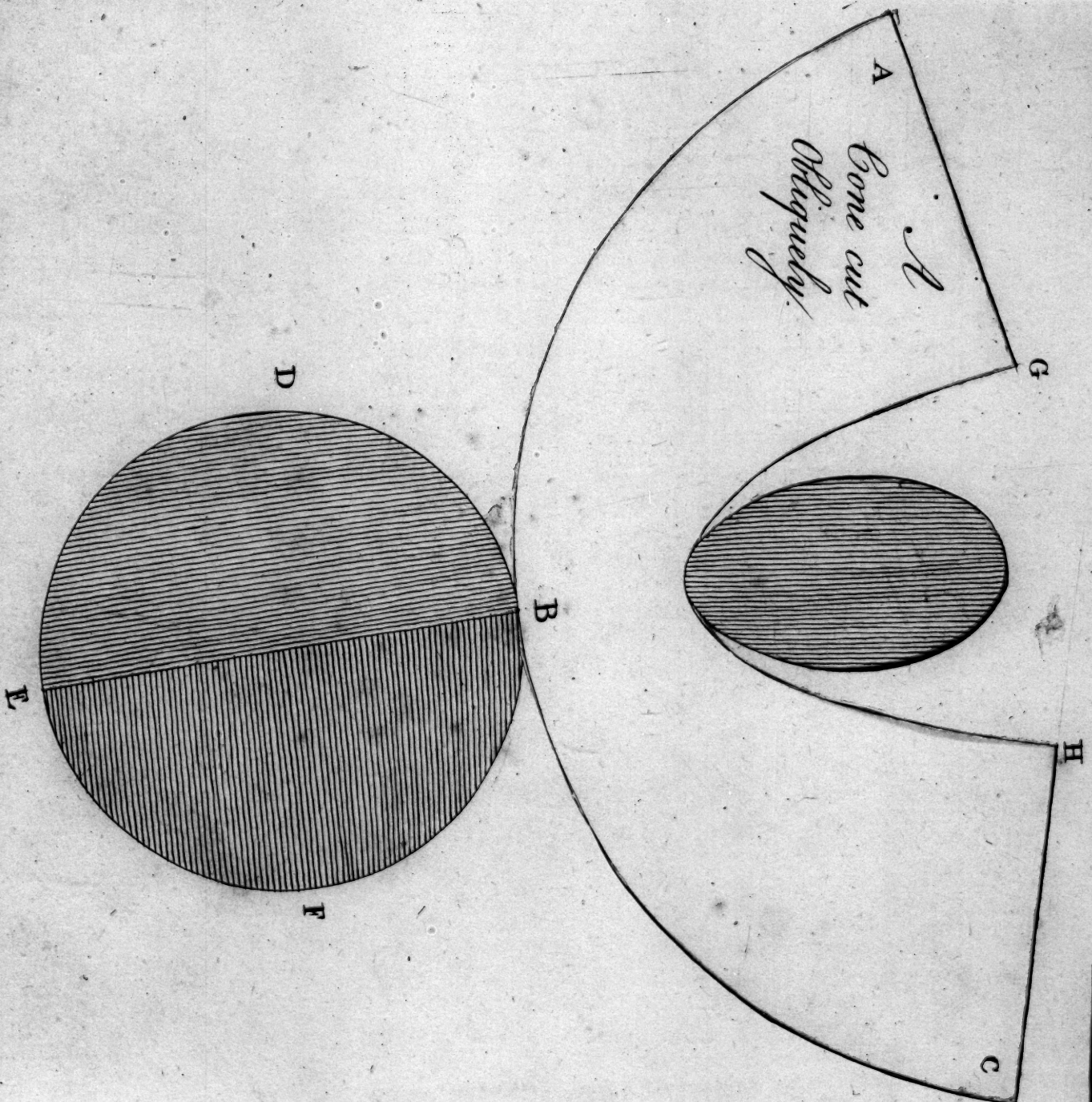


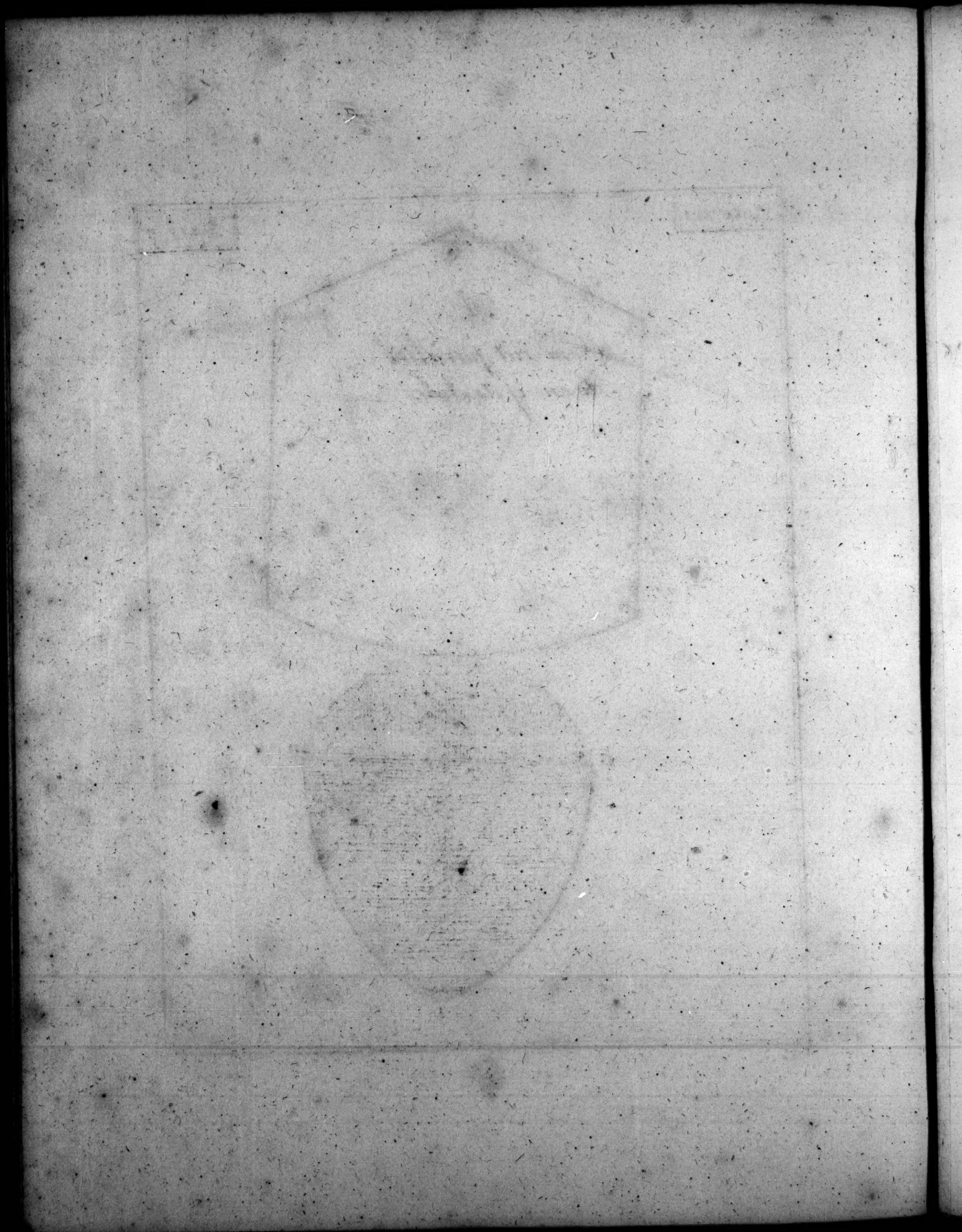
Eucl. 12. Prop. 7.

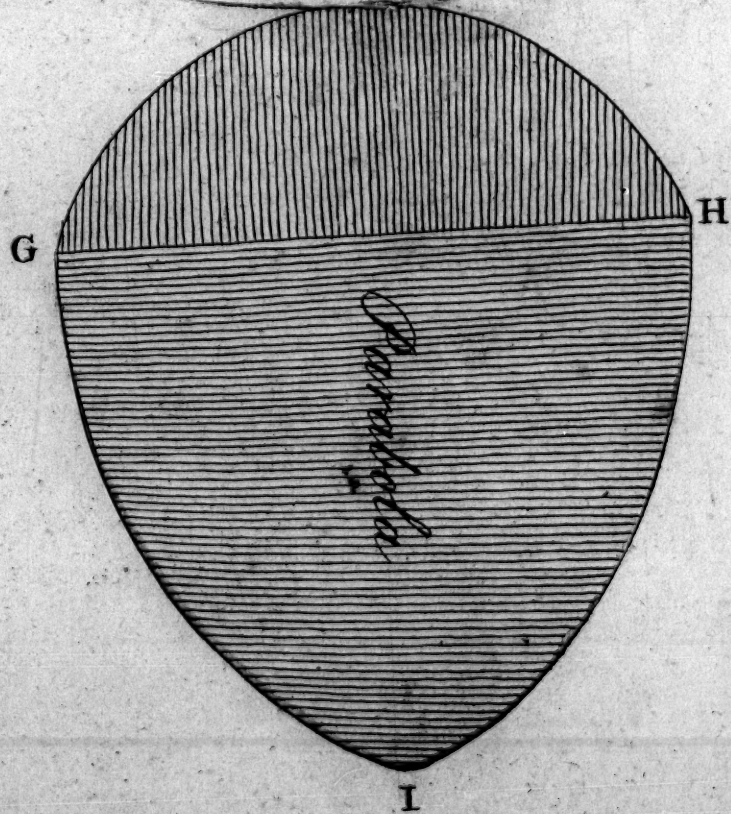
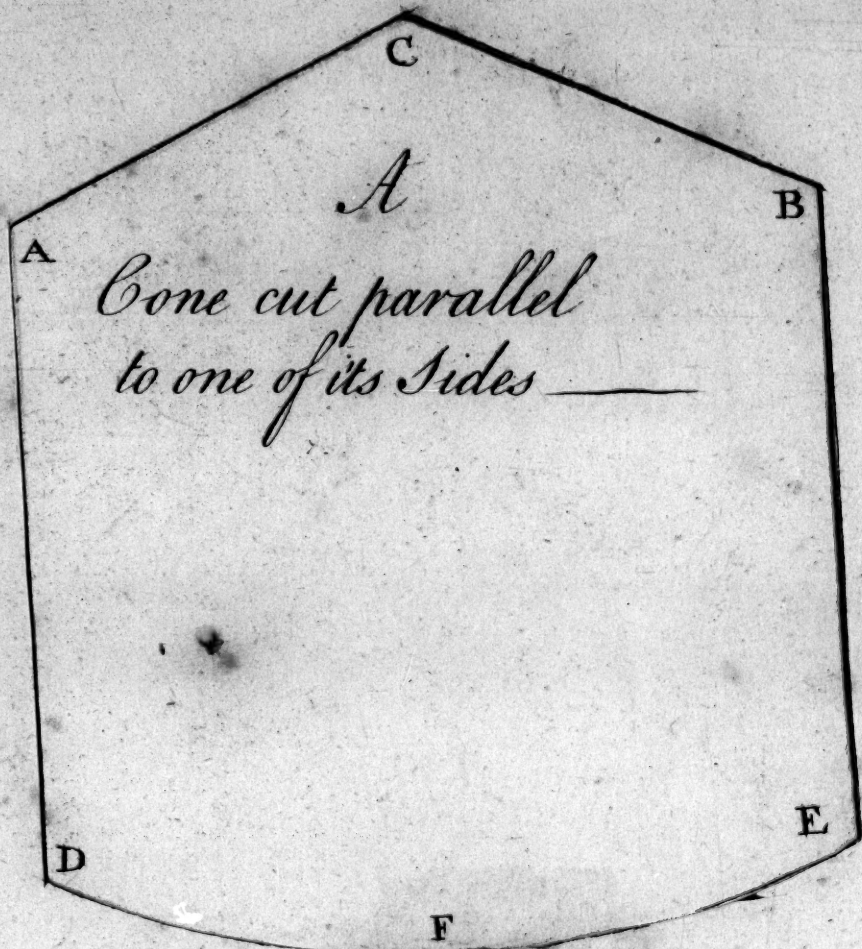












A
Cone cut
parallel to its axis.

